JHMT 2013 Geometry Test
February 2, 2013

Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written on the answer sheet will be considered for grading.
No calculators.

Problem 1. In triangle ABC, $AC = 7$. D lies on AB such that $AD = BD = CD = 5$. Find BC.

Problem 2. Robin has obtained a circular pizza with radius 2. However, being rebellious, instead of slicing the pizza radially, he decides to slice the pizza into 4 strips of equal width both vertically and horizontally. What is the area of the smallest piece of pizza?

Problem 3. A circle of radius 2 is inscribed in equilateral triangle ABC. The altitude from A to BC intersects the circle at a point D not on BC. BD intersects the circle at a point E distinct from D. Find the length of BE.

Problem 4. Points A, B, and C lie on a circle of radius 5 such that $AB = 6$ and $AC = 8$. Find the smaller of the two possible values of BC.

Problem 5. In square $ABCD$ with side length 2, let P and Q be on side AB such that $AP = BQ = \frac{1}{2}$. Let E be a point on the square that maximizes the angle PEQ. Find the area of triangle PEQ.

Problem 6. $ABCD$ is a rectangle with $AB = CD = 2$. A circle centered at O is tangent to BC, CD, and AD (and hence has radius 1). Another circle, centered at P, is tangent to circle O at point T and is also tangent to AB and BC. If line AT is tangent to both circles at T, find the radius of circle P.

Problem 7. $ABCD$ is a square such that $\overline{AB}$ lies on the line $y = x + 4$ and points C and D lie on the graph of parabola $y^2 = x$. Compute the sum of all possible areas of $ABCD$.

Problem 8. After weeks of toil in the lab, Randall has finally made for himself a hollow clear plastic sphere of radius 1 m. Initially deciding that he wants to use it as a giant die, he draws six dots on the surface of the sphere, each in a different color, so that the six dots are vertices of a regular octahedron. Later, he changes his mind, and decides to use it as a giant swimming pool instead. He cuts a hole in the top and starts flooding the inside with water, holding the sphere still. Find the smallest amount of water, in cubic meters, that he would have to use in order to guarantee that at least one of the dots lies at or below the water level.

Problem 9. Let equilateral triangle ABC with side length 6 be inscribed in a circle and let P be on arc AC such that $AP \cdot PC = 10$. Find the length of BP.

Problem 10. In tetrahedron $ABCD$, $AB = 4$, $CD = 7$, and $AC = AD = BC = BD = 5$. Let I_A, I_B, I_C, and I_D denote the incenters of the faces opposite vertices A, B, C, and D, respecitvely. It is provable that AI_A intersects BI_B at a point X, and CI_C intersects DI_D at a point Y. Compute XY.