JHMT 2013 General Test 2
February 2, 2013

Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written on the answer sheet will be considered for grading.
No calculators.

Problem 1. Ben is throwing darts at a circular target with diameter 10. Ben never misses the target when he throws a dart, but he is equally likely to hit any point on the target. Ben gets $\lceil 5 - x \rceil$ points for having the dart land x units away from the center of the target. What is the expected number of points that Ben can earn from throwing a single dart?

Problem 2. Consider a sequence given by $a_n = a_{n-1} + 3a_{n-2} + a_{n-3}$, where $a_0 = a_1 = a_2 = 1$. What is the remainder of a_{2013} when divided by 7?

Problem 3. What is the smallest number over 9000 that is divisible by the first four primes?

Problem 4. Circles A and B both have radius 1 and pass through each other's centers. What is the area of the union of the two circles?

Problem 5. Let $a = 0, b = 1, \ldots, z = 25, Z = 26, Y = 27, \ldots, A = 51$ be the digits of a base 52 number system. Calculate $A \times z$ and express your answer in base 52.

Problem 6. A triangle with side lengths 2 and 3 has an area of 3. Compute the third side length of the triangle.

Problem 7. A tree has 10 pounds of apples at dawn. Every afternoon, a bird comes and eats x pounds of apples. Overnight, the amount of food on the tree increases by 10%. What is the maximum value of x such that the bird can sustain itself indefinitely on the tree without the tree running out of food?

Problem 8. $\mathbb{R}^2$-tic-tac-toe is a game where two players take turns putting red and blue points anywhere on the xy plane. The red player moves first. The first player to get 3 of their points in a line without any of their opponent's points in between wins. What is the least number of moves in which red can guarantee a win? (We count each time that red places a point as a move, including when red places their winning point.)

Problem 9. Andrew flips a fair coin 5 times, and counts the number of heads that appear. Beth flips a fair coin 6 times and also counts the number of heads that appear. Compute the probability Andrew counts at least as many heads as Beth.

Problem 10. An unfair coin lands heads with probability $\frac{1}{17}$ and tails with probability $\frac{16}{17}$. Matt flips the coin repeatedly until he flips at least one head and at least one tail. What is the expected number of times that Matt flips the coin?