

JHMT 2013 General Test 1
February 2, 2013

Time limit: 50 minutes.

Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written on the answer sheet will be considered for grading.

No calculators.

Problem 1. Robin goes birdwatching one day. He sees three types of birds: penguins, pigeons, and robins. $\frac{2}{3}$ of the birds he sees are robins. $\frac{1}{8}$ of the birds he sees are penguins. He sees exactly 5 pigeons. How many robins does Robin see?

Problem 2. A rhombus has area 36 and the longer diagonal is twice as long as the shorter diagonal. What is the perimeter of the rhombus?

Problem 3. A fly and an ant are on one corner of a unit cube. They wish to head to the opposite corner of the cube. The fly can fly through the interior of the cube, while the ant has to walk across the faces of the cube. How much shorter is the fly's path if both insects take the shortest path possible?

Problem 4. Steve is writing an essay for class. He takes 90 minutes to write a 1000-word essay. Steve gets tired of writing essays though - if the number of words he needs to write for an essay doubles, then he needs to take four times as long to write that essay. If Steve has a whole day to write an essay, what is the maximum number of words that can be in his essay?

Problem 5. Given regular hexagon $ABCDEF$, compute the probability that a randomly chosen point inside the hexagon is inside triangle PQR , where P is the midpoint of AB , Q is the midpoint of CD , and R is the midpoint of EF .

Problem 6. Whenever Moor Xu walks into an auditorium, people scream his name. In one particular auditorium, the screaming has a volume of $f(t) = 100 + 50 \sin(\pi t)$ decibels at second t . Moor can't hear himself think unless the volume is lower than 75 decibels. Moor runs out of the auditorium after one minute. For how many seconds could Moor not hear himself think?

Problem 7. An isosceles right triangle is inscribed in a circle with radius 5, thereby separating the circle into four regions. Compute the sum of the areas of the two smallest regions.

Problem 8. Compute the square of the distance between the incenter (center of the inscribed circle) and circumcenter (center of the circumscribed circle) of a 30-60-90 right triangle with hypotenuse of length 2.

Problem 9. Nick has a terrible sleep schedule. He randomly picks a time between 4 AM and 6 AM to fall asleep, and wakes up at a random time between 11 AM and 1 PM of the same day. What is the probability that Nick gets at least 6 hours of sleep?

Problem 10. A positive integer $b \geq 2$ is *neat* if and only if there exist positive base- b digits x and y (that is, x and y are integers and $0 < x, y < b$) such that the number $x.y$ base b (that is, $x + \frac{y}{b}$) is an integer multiple of x/y . Find the number of *neat* integers less than or equal to 100.