

JHMT 2013 Calculus Test
February 2, 2013

Time limit: 50 minutes.

Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written on the answer sheet will be considered for grading.

No calculators.

Problem 1. Compute $\lim_{x \rightarrow 3} \frac{x^2 + 2x - 15}{x^2 - 4x + 3}$.

Problem 2. Compute all real values of b such that, for $f(x) = x^2 + bx - 17$, $f(4) = f'(4)$.

Problem 3. Ian the ant is travelling on the Cartesian coordinate plane. He wishes to travel from his current location, the point $(0, 0)$ to the point $(4, 8)$. However, due to an unusual fear of straight lines, Ian decides to make his journey along the curve defined by $y = x^{3/2}$. Assuming Ian goes directly from his starting point to his ending point, calculate the distance travelled on his journey.

Problem 4. Let $f_n(x) = \frac{\lfloor nx \rfloor}{n}$ where n is a positive integer and let $g(x) = \lim_{n \rightarrow \infty} f_n(x)$ for any given x . What is $\int_{-3}^3 g(x) dx$?

Problem 5. Let $f_0(x) = xe^x$, and for $n \geq 1$, define $f_n(x) = \int_n^x f_{n-1}(t) dt$. Compute $f_{2012}(2013)$.

Problem 6. Compute

$$\int_0^1 \frac{\log(1-x)}{x} dx.$$

Problem 7. Evaluate

$$\int_0^1 \lceil \sin(\pi/x) \rceil dx.$$

Problem 8. Compute

$$\int_0^{\pi/2} \frac{dx}{\left(\sqrt{\sin(x)} + \sqrt{\cos(x)}\right)^4}.$$

Problem 9. Evaluate

$$\lim_{n \rightarrow \infty} \prod_{k=1}^n \frac{2k}{2k-1} \int_{-1}^{\infty} \frac{(\cos x)^{2n}}{2^x} dx.$$

Problem 10. Suppose we climb a mountain that is a cone with radius $r_0 = 100$ and height $h = 4$. We start at the bottom of the mountain (on the perimeter of the base of the cone), and our destination is the opposite side of the mountain, halfway up (height $h = 2$). Our climbing speed starts at $v_0 = 2$ but gets slower at a rate inversely proportional to the distance to the mountain top (so at height z the speed is $(h - z)v_0/h$). Find the minimum time needed to get to the destination.