JHMT 2013 Algebra Test
February 2, 2013

Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written on the answer sheet will be considered for grading.
No calculators.

Problem 1. Nick is a runner, and his goal is to complete four laps around a circuit at an average speed of 10 mph. If he completes the first three laps at a constant speed of only 9 mph, what speed does he need to maintain on the fourth lap to achieve his goal?

Problem 2. Compute the largest root of $x^4 - x^3 - 5x^2 + 2x + 6$.

Problem 3. Karl likes the number 17. His favorite polynomials are monic quadratics with integer coefficients such that 17 is a root of the quadratic and the roots differ by no more than 17. Compute the sum of the coefficients of all of Karl's favorite polynomials. (A monic polynomial is a polynomial with a leading coefficient of 1.)

Problem 4. The parabola $x^2 + 2x + 3$ is tangent to the line $y = bx - 17$ for exactly two real values of b, namely b_1 and b_2. Compute $b_1^2 + b_2^2$.

Problem 5. Find all real x that satisfy $\sqrt[3]{20x + \sqrt[3]{20x + 13}} = 13$.

Problem 6. What is the greatest possible value of c such that $x^2 + 5x + c = 0$ has two (not necessarily distinct) real roots?

Problem 7. Given a complex number z such that $z^{13} = 1$, find all possible values of $z + z^3 + z^4 + z^9 + z^{10} + z^{12}$.

Problem 8. Find the sum of all real x such that
$$\frac{4x^2 + 15x + 17}{x^2 + 4x + 12} = \frac{5x^2 + 16x + 18}{2x^2 + 5x + 13}.$$

Problem 9. Rationalize the denominator of
$$\frac{1}{2\sqrt[3]{4} + \sqrt[3]{2} + 1}.$$
(In other words, write it in the form $\frac{a\sqrt[3]{4} + b\sqrt[3]{2} + c}{d}$ where a, b, c, d are integers.)

Problem 10. Given real numbers x, y, z that satisfy $x + 3y + 2z = 1$, find the minimum value of $3x^2 - y^2 + 2z^2$.