Time limit: 1 hour.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written on the answer sheet will be considered for grading.
No calculators.

1. A standard 12-hour clock has hour, minute, and second hands. How many times do two hands cross between 1:00 and 2:00 (not including 1:00 and 2:00 themselves)?

2. Define a set of positive integers to be *balanced* if the set is not empty and the number of even integers in the set is equal to the number of odd integers in the set. How many strict subsets of the set of the first 10 positive integers are balanced?

3. How many ordered sequences of 1's and 3's sum to 16? (Examples of such sequences are $\{1, 3, 3, 3, 3, 3\}$ and $\{1, 3, 1, 3, 1, 3, 1, 3\}$.)

4. How many positive numbers up to and including 2012 have no repeating digits?

5. Define a number to be *boring* if all the digits of the number are the same. How many positive integers less than 10000 are both prime and boring?

6. A permutation of the first n positive integers is quadratic if, for some positive integers a and b such that $a + b = n$, $a \neq 1$, and $b \neq 1$, the first a integers of the permutation form an increasing sequence and the last b integers of the permutation form a decreasing sequence, or if the first a integers of the permutation form a decreasing sequence and the last b integers of the permutation form an increasing sequence. How many permutations of the first 10 positive integers are quadratic?

7. Two different squares are randomly chosen from an 8×8 chessboard. What is the probability that two queens placed on the two squares can attack each other? Recall that queens in chess can attack any square in a straight line vertically, horizontally, or diagonally from their current position.

8. A short rectangular table has four legs, each 8 inches long. For each leg Bill picks a random integer x, $0 \leq x < 8$ and cuts x inches off the bottom of that leg. After he's cut all four legs, compute the probability that the table won't wobble (i.e. that the ends of the legs are coplanar).

9. Two ants are on opposite vertices of a regular octahedron (an 8-sized polyhedron with 6 vertices, each of which is adjacent to 4 others), and make moves simultaneously and continuously until they meet. At every move, each ant randomly chooses one of the four adjacent vertices to move to. Eventually, they will meet either at a vertex (that is, at the completion of a move) or on an edge (that is, in the middle of a move). Find the probability that they meet on an edge.

10. We say that two polynomials $F(x)$ and $G(x)$ are equivalent mod 5 if and only if $F(x) - G(x) = 5 \cdot H(x)$ for some integer polynomial $H(x)$. We say that $F(x)$ has n as a root mod 5 if and only if $5 \mid F(n)$. How many inequivalent integer polynomials mod 5 of degree at most 3 do not have any integer roots mod 5?