

Time limit: 1 hour.

Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written on the answer sheet will be considered for grading.

No calculators.

1. Define a set of positive integers to be *balanced* if the set is not empty and the number of even integers in the set is equal to the number of odd integers in the set. How many strict subsets of the set of the first 10 positive integers are balanced?
2. At the 2012 Silly Math Tournament, hamburgers and hot dogs are served. Each hamburger costs \$4 and each hot dog costs \$3. Each team has between 6 and 10 members, inclusive, and each member buys exactly one food item. How many different values are possible for a team's total food cost?
3. How many ordered sequences of 1's and 3's sum to 16? (Examples of such sequences are $\{1, 3, 3, 3, 3, 3\}$ and $\{1, 3, 1, 3, 1, 3, 1, 3\}$.)
4. How many positive numbers up to and including 2012 have no repeating digits?
5. ABC is an equilateral triangle with side length 1. Point D lies on \overline{AB} , point E lies on \overline{AC} , and points G and F lie on \overline{BC} , such that $DEFG$ is a square. What is the area of $DEFG$?
6. If f is a monic cubic polynomial with $f(0) = -64$, and all roots of f are non-negative real numbers, what is the largest possible value of $f(-1)$? (A polynomial is monic if it has a leading coefficient of 1.)
7. In trapezoid $ABCD$, $BC \parallel AD$, $AB = 13$, $BC = 15$, $CD = 14$, and $DA = 30$. Find the area of $ABCD$.
8. Circle O has radius 18. From diameter AB , there exists a point C such that BC is tangent to O and AC intersects O at a point D , with $AD = 24$. What is the length of BC ?
9. The quartic (4th-degree) polynomial $P(x)$ satisfies $P(1) = 0$ and attains its maximum value of 3 at both $x = 2$ and $x = 3$. Compute $P(5)$.
10. Compute the ordered pair of real numbers (a, b) such that $a < k < b$ if and only if $x^3 + \frac{1}{x^3} = k$ does not have a real solution in x .