1. In preparation for the annual USA Cow Olympics, Bessie is undergoing a new training regime. However, she has procrastinated on training for too long, and now she only has exactly three weeks to train. Bessie has decided to train for 45 hours. She spends a third of the time training during the second week as she did during the first week, and she spends a half of the time training during the third week as during the second week. How much time did she spend training during the second week?

   Answer: 10

   Solution: Let x be the number of hours spent training during the second week. We have that $3x + x + \frac{x}{2} = 45$. Therefore, $x = \boxed{10}$.

2. The Tribonacci numbers T_n are defined as follows: $T_0 = 0$, $T_1 = 1$, and $T_2 = 1$. For all $n \geq 3$, we have $T_n = T_{n-1} + T_{n-2} + T_{n-3}$. Compute the smallest Tribonacci number greater than 100 which is prime.

   Answer: 149

   Solution: The first few Tribonacci numbers are 0, 1, 1, 2, 4, 7, 13, 24, 44, 81, 149. $\boxed{149}$ is the smallest Tribonacci number greater than 100, and it also turns out to be prime, so that is our answer.

3. Steve works 40 hours a week at his new job. He usually gets paid 8 dollars an hour, but if he works for more than 8 hours on a given day, he earns 12 dollars an hour for every additional hour over 8 hours. If x is the maximum number of dollars that Steve can earn in one week by working exactly 40 hours, and y is the minimum number of dollars that Steve can earn in one week by working exactly 40 hours, what is $x - y$?

   Answer: 96

   Solution: In the minimum case, Steve can work 8 hours a day for five days, thereby earning no overtime pay and earning exactly \$320. In the maximum case, Steve works 40 hours without a single break. This spans two days; there are 16 hours of work at regular pay and 24 hours of work at overtime pay. Therefore, Steve earns $16 \times \$8 + 24 \times \$12 = \$416$. Therefore, $x = 416$ and $y = 320$, so $x - y = \boxed{96}$.

4. There are 100 people in a room. 60 of them claim to be good at math, but only 50 are actually good at math. If 30 of them correctly deny that they are good at math, how many people are good at math but refuse to admit it?

   Answer: 10

   Solution: By the principle of inclusion and exclusion, the sum of the number of people who are good at math and the number of people who claim to be good at math minus the number of people in both categories gives the number of people who either are good at math or claim they are good at math. Let x be the number of people in both categories. Then $50 + 60 - x = 100 - 30$, so $x = 40$. Thus we are left with $50 - 40 = \boxed{10}$ people who are good at math but refuse to admit it.

5. A standard 12-hour clock has hour, minute, and second hands. How many times do two hands cross between 1:00 and 2:00 (not including 1:00 and 2:00 themselves)?

Answer: 119

Solution: We know that the hour and minute hands cross exactly once. Let m be the number of minutes past one o'clock that this happens. The angle between the minute hand and the 12 must be equal to the angle between the hour hand and the 12. Since 1 minute is $\frac{360^\circ}{60} = 6^\circ$ on the clock and 1 hour is $\frac{360^\circ}{12} = 30^\circ$, we have $6m = 30(1 + \frac{m}{60})$, so $m = \frac{60}{11} = 5\frac{5}{11}$. Note that the second hand is not at the same position at this time, so we do not have to worry about a triple crossing.

On the other hand, the second hand crosses the hour hand once every minute, for a total of 60 crossings. Also, the second hand crosses the minute hand once every minute except the first and last, since those crossings take place at 1:00 and 2:00, for a total of 58 crossings. There is a grand total of $1 + 60 + 58 = \boxed{119}$ crossings.

6. Define a number to be *boring* if all the digits of the number are the same. How many positive integers less than 10000 are both prime and boring?

   Answer: 5

   Solution: The one-digit boring primes are 2, 3, 5, and 7. The only two-digit boring prime is 11, since 11 divides all other two-digit boring numbers. No three-digit boring numbers are prime, since 111 divides all of them and $111 = 3 \times 37$. No four-digit boring numbers are prime since they are all divisible by 11. Therefore, there are $\boxed{5}$ positive integers less than 10000 which are both prime and boring.

7. Given a number n in base 10, let $g(n)$ be the base-3 representation of n. Let $f(n)$ be equal to the base-10 number obtained by interpreting $g(n)$ in base 10. Compute the smallest positive integer $k \geq 3$ that divides $f(k)$.

   Answer: 7

   Solution: Using brute force, we note that 3, 4, 5, and 6 are invalid, but $7 = 21_3$. Thus, the answer is $\boxed{7}$.

8. $ABCD$ is a parallelogram. $AB = BC = 12$, and $\angle ABC = 120^\circ$. Calculate the area of parallelogram $ABCD$.

   Answer: $72\sqrt{3}$

   Solution: Since opposite sides of a parallelogram are equal, $AB = BC = CD = DA = 12$. Since adjacent angles of a parallelogram are supplementary, $\angle BCD = \angle CDA = 60^\circ$. Therefore, when we draw diagonal BD, we get two equilateral triangles, both with side length 12. The area of an equilateral triangle with side length s is $\dfrac{s^2\sqrt{3}}{4}$, so therefore the area of the parallelogram is $2 \times \dfrac{12^2 \times \sqrt{3}}{4} = \boxed{72\sqrt{3}}$.

9. Given a 1962-digit number that is divisible by 9, let x be the sum of its digits. Let the sum of the digits of x be y. Let the sum of the digits of y be z. Compute the maximum possible value of z.

   Answer: 9

Solution: Let the 1962 digit number be a. First of all note that $9 \mid x, y, z$. This is because $a = \sum_{i=1}^{1962} a_i 10^i \equiv \sum_{i=1}^{1962} a_i \mod 9 = x$. Now clearly the largest value of x would be obtained if $a = \sum_{i=1}^{1962} 9(10)^i$, hence $x \leq 1962(9) = 17658$. It follows that, $y \leq 1+7+6+5+8 = 36$ and finally that $z \leq 9$. However $9 \mid z$ so $z = \boxed{9}$.

10. A circle with radius 1 has diameter AB. C lies on this circle such that $\overset{\frown}{AC}$ / $\overset{\frown}{BC}= 4$. $\overline{AC}$ divides the circle into two parts, and we will label the smaller part Region I. Similarly, $\overline{BC}$ also divides the circle into two parts, and we will denote the smaller one as Region II. Find the positive difference between the areas of Regions I and II.

Answer: $\frac{3\pi}{10}$

Solution: Let O be the center of the circle. Note that CO bisects AB, so the areas of $\triangle ACO$ and $\triangle BCO$ are equal. Hence, the desired difference in segment areas is equal to the difference in the areas of the corresponding sectors. The sector corresponding to $\overset{\frown}{AC}$ has area $\frac{2\pi}{5}$, and the sector corresponding to $\overset{\frown}{BC}$ has area $\frac{\pi}{10}$, so the desired difference is $\boxed{\dfrac{3\pi}{10}}$.