Time limit: 1 hour.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written on the answer sheet will be considered for grading.
No calculators.

1. Compute the minimum possible value of

$$(x - 1)^2 + (x - 2)^2 + (x - 3)^2 + (x - 4)^2 + (x - 5)^2,$$

   for real values of x.

2. Express $\frac{2^3-1}{2^3+1} \times \frac{3^3-1}{3^3+1} \times \frac{4^3-1}{4^3+1} \times \cdots \times \frac{16^3-1}{16^3+1}$ as a fraction in lowest terms.

3. If x, y, and z are integers satisfying $xyz + 4(x + y + z) = 2(xy + xz + yz) + 7$, list all possibilities for the ordered triple (x, y, z).

4. The quartic (4th-degree) polynomial $P(x)$ satisfies $P(1) = 0$ and attains its maximum value of 3 at both $x = 2$ and $x = 3$. Compute $P(5)$.

5. Compute the ordered pair of real numbers (a, b) such that $a < k < b$ if and only if $x^3 + \dfrac{1}{x^3} = k$ does not have a real solution in x.

6. If f is a monic cubic polynomial with $f(0) = -64$, and all roots of f are non-negative real numbers, what is the largest possible value of $f(-1)$? (A polynomial is monic if it has a leading coefficient of 1.)

7. There exist two triples of real numbers (a, b, c) such that $a - \frac{1}{b}$, $b - \frac{1}{c}$, and $c - \frac{1}{a}$ are the roots to the cubic equation $x^3 - 5x^2 - 15x + 3$ listed in increasing order. Denote those (a_1, b_1, c_1) and (a_2, b_2, c_2). If a_1, b_1, and c_1 are the roots to monic cubic polynomial f and a_2, b_2, and c_2 are the roots to monic cubic polynomial g, find $f(0)^3 + g(0)^3$.

8. How many positive integers n are there such that for any natural numbers a, b, we have $n \mid (a^2b + 1)$ implies $n \mid (a^2 + b)$? (Note: The symbol $\mid$ means "divides"; if $x \mid y$ then y is a multiple of x.)

9. The function $f(x)$ is known to be of the form $\prod_{i=1}^{n} f_i(a_i x)$, where a_i is a real number and $f_i(x)$ is either $\sin(x)$ or $\cos(x)$ for $i = 1, \ldots, n$. Additionally, $f(x)$ is known to have zeros at every integer between 1 and 2012 (inclusive) except for one integer b. Find the sum of all possible values of b.

10. For real numbers (x, y, z) satisfying the following equations, find all possible values of $x + y + z$.

$$x^2 y + y^2 z + z^2 x = -1$$
$$xy^2 + yz^2 + zx^2 = 5$$
$$xyz = -2$$