# 12$^{\text{th}}$ Annual Johns Hopkins Math Tournament
Saturday, February 19, 2011

Combinatorics/Probability Subject Test

1. [**1025**] Natasha flips 10 fair coins and counts the number of heads. What is the probability that Natasha flipped 5 heads and 5 tails?

2. [**1026**] Find the number of pairs (a, b) with a, b positive integers such that $\frac{a}{b}$ is in lowest terms and $a + b \leq 10$.

3. [**1028**] An *icosahedron* is a regular polyhedron with 12 vertices, 20 faces, and 30 edges. How many rigid rotations G are there for an icosahedron in $\mathbb{R}^3$?

4. [**1032**] You are standing at the base of a staircase with 11 steps. At any point, you are allowed to move either 1 step up or 2 steps up. How many ways are there for you to reach the top step?

5. [**1040**] Mordecai is standing in front of a 100-story building with two identical glass orbs. He wishes to know the highest floor from which he can drop an orb without it breaking. What is the minimum number of drops Mordecai can make such that he knows for certain which floor is the highest possible?

6. [**1056**] Two ants, Yuri and Jiawang, begin on opposite corners of a cube. On each move, they can travel along an edge to an adjacent vertex. Find the probability they both return to their starting position after 4 moves.

7. [**1088**] If a rectangle is drawn in a 2011×2011 square grid (degenerate rectangles do not count), what is the expected value of the area of the rectangle?

8. [**1152**] Consider the following 5-by-5 square and 3-by-1 rectangle:
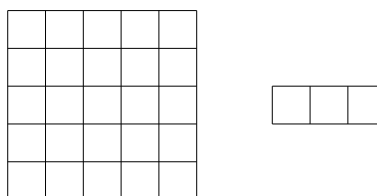
Define a *tiling* of the square by the rectangle to be a configuration in which eight nonoverlapping 3-by-1 rectangles are placed inside the 5-by-5 square, possibly rotated by 90 degrees but with grid lines matching up, with only one subsquare of the 5-by-5 square remaining uncovered. Find the number of such tilings, counting rotations and reflections as distinct.

9. [**1280**] Determine the maximum number of ways that 10 circles and 10 lines can divide the plane into disjoint regions.

10. [**1536**] How many functions f that take $\{1, 2, 3, 4, 5\}$ to itself, not necessarily injective or surjective, satisfy $f(f(f(x))) = f(f(x))$ for all x in $\{1, 2, 3, 4, 5\}$?