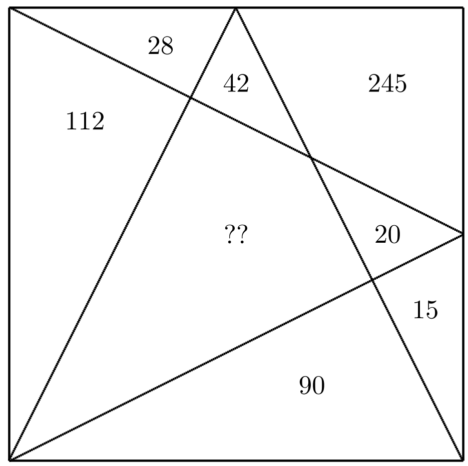
11^{th} Annual Johns Hopkins Math Tournament
Sunday, April 11, 2010
Grab Bag-Lower Division

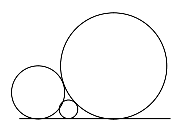
(1) (**6**) Below is a square, divided by several lines (not to scale). Several regions have their areas written inside. Find the area of the remaining region.



(2) (**7**) Three unit circles ω_1, ω_2, and ω_3 in the plane have the property that each circle passes through the centers of the other two. Find the area of the region that is intersected by all of ω_1, ω_2, and ω_3.

(3) (**8**) Two integers are called *relatively prime* if they share no common prime factors (that is, their greatest common divisor is 1). Given that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}$, find the probability that two integers picked at random are relatively prime.

(4) (**10**) Two circles with radii 1 and 2 are arranged in such a way that they are tangent to each other and a straight line. A third circle is placed between them so that it is tangent to both circles and the line, as shown in the diagram below (not drawn to scale). Find the radius of the small circle.



(5) (**11**) A box contains 4 green balls, 4 blue balls, 2 red balls, a yellow ball, a white ball, and a black ball. The balls are picked randomly, one at a time without replacement, until two balls of the same color have been removed. The process requires that at most 7 balls be removed. Find the probability that 7 balls are removed. You may leave your answer in terms of $\binom{n}{k}$ or as an explicit probability.

(6) (**13**) Jason, Ben, Yichen, and Anthony are playing Contract Bridge. After one hand, they notice that all of the cards of two suits are split between Jason and Ben. Determine the number of ways 13 cards can be dealt to each player such that this is the case. You may leave your answer in $\binom{n}{k}$ form. (Contract Bridge is played with a standard 52-card deck.)

(7) (**14**) Fifteen chairs are lined up in a row for Professor Zucker's Honors Linear Algebra Exam. However, only 6 students show up and Zucker won't let any two students sit next to each other. In how many ways can Zucker arrange his students?

(8) (**15**) A point P is randomly placed on $\overline{AB}$. Let M be the midpoint of $\overline{AB}$. What is the probability that that AP, PB, and AM can be made to form a triangle? That is, what is the probability that AP, PB, and AM are possible lengths for a triangle?

(9) (**16**) Let $\triangle ABC$, be acute with perimeter 100. Let D be a point on $\overline{BC}$. The circumcircles of ABD and ADC intersect $\overline{AC}$ and $\overline{AB}$ at E and F respectively such that $DE = 14$ and $DF = 11$. If $\angle EBC \cong \angle BCF$, find $\frac{AE}{AF}$.