

Johns Hopkins Mathematics Tournament

April 8, 2006

TEAM QUESTION PAPER

1. Evaluate S .

$$S = \frac{10000^2 - 1}{\sqrt{10000^2 - 19999}}.$$

2. Starting on a triangular face of a right triangular prism and allowing moves to only adjacent faces, how many ways can you pass through each of the other four faces and return to the first face in five moves?

3. Given that

$$\begin{aligned}(a + b) + (b + c) + (c + a) &= 18, \\ \frac{1}{a + b} + \frac{1}{b + c} + \frac{1}{c + a} &= \frac{5}{9},\end{aligned}$$

determine

$$\frac{c}{a + b} + \frac{a}{b + c} + \frac{b}{c + a}.$$

4. Find all primes p such that $2^{p+1} + p^3 - p^2 - p$ is prime.
5. In right triangle ABC with the right angle at A , \overline{AF} is the median, \overline{AH} is the altitude, and \overline{AE} is the angle bisector. If $\angle EAF = 30^\circ$, find $\angle BAH$ in degrees.
6. For which integers a does the equation

$$(1 - a)(a - x)(x - 1) = ax$$

not have two distinct real roots of x ?

7. Given that

$$a^2 + b^2 - ab - b + \frac{1}{3} = 0,$$

solve for all (a, b) .

8. Point E is on side \overline{AB} of the unit square $ABCD$. F is chosen on \overline{BC} so that $AE = BF$, and G is the intersection of \overline{DE} and \overline{AF} . As the location of E varies along side \overline{AB} , what is the minimum length of \overline{BG} ?
9. Sam and Susan are taking turns shooting a basketball. Sam goes first and has probability P of missing any shot, while Susan has probability P of making any shot. What must P be so that Susan has a 50% chance of making the first shot?
10. Quadrilateral $ABCD$ has $AB = BC = CD = 7$, $AD = 13$, $\angle BCD = 2\angle DAB$, and $\angle ABC = 2\angle CDA$. Find its area.