

Johns Hopkins Mathematics Tournament

April 8, 2006

GEOMETRY QUESTION PAPER

1. Unit square $ZINC$ is constructed in the interior of hexagon $CARBON$. What is the area of triangle BIO ?
2. If two altitudes of a triangle have length 12 and 4, what integral lengths can the third altitude attain?
3. Rectangle $ABCD$ is folded in half so that the vertices D and B coincide, creating the crease \overline{EF} , with E on \overline{AD} and F on \overline{BC} . Let O be the midpoint of \overline{EF} . If triangles DOC and DCF are congruent, what is the ratio $BC : CD$?
4. The square $DEFG$ is contained in equilateral triangle ABC , with E on \overline{AC} , G on \overline{AD} , and F as the midpoint of \overline{BC} . Find AD if $DE = 6$.
5. An ant is on the bottom edge of a right circular cone with base area π and slant length 6. What is the shortest distance that the ant has to travel to loop around the cone and come back to its starting position?
6. A right cylinder is inscribed in a right circular cone with height 2 and radius 2 so that the cylinder's bottom base sits on the cone's base. What is the maximum possible surface area of the cylinder?
7. AD is the angle bisector of the right triangle ABC with $\angle ABC = 60^\circ$ and $\angle BCA = 90^\circ$. E is chosen on \overline{AB} so that the line parallel to \overline{DE} through C bisects \overline{AE} . Find $\angle EDB$ in degrees.
8. Circles P , Q , and R are externally tangent to one another. The external tangent of P and Q that does not intersect R intersects P and Q at P_Q and Q_P , respectively. Q_R , R_Q , R_P , and P_R are defined similarly. If the radius of Q is 4 and $\overline{Q_P P_Q} \parallel \overline{R_Q Q_R}$, compute $R_P P_R$.