

Johns Hopkins Mathematics Tournament

April 8, 2006

ALGEBRA QUESTION PAPER

1. Determine the units digit of $87^{65} + 43^{21}$.
2. How many integer pairs (a, b) for $-5 \leq a, b \leq 5$ are there such that $ax^2 + 10x + b = 0$ has two distinct real roots?
3. Find all x in $[0, \pi]$ inclusive so that

$$\sin^2 x + \csc^2 x + \cos^2 x = \cot^2 x + \sec^2 x.$$

4. Solve for all values of x .

$$x^{\log x} = 1000x^2.$$

5. Given that the number $m3mmmmmm$ is a prime for at least one digit m , find all such m .
6. A circular disk of gradually decreasing radius slides down a pit defined by the equation $y = x^2$, maintaining two points of contact with the pit. What is its radius when the two points of contact coalesce into one?
7. Between which two consecutive integers does S lie?

$$S = \sqrt{18 + \sqrt{18^2 + \sqrt{18^3 + \dots}}}$$

8. If the range of

$$f(x, y, z) = \frac{x}{x+y} + \frac{y}{y+z} + \frac{z}{z+x}$$

for positive x, y , and z is (A, B) exclusive, find $A + B$.