1. Consider the following function:

\[ f(x) = \left( \frac{1}{2} \right)^x - \left( \frac{1}{2} \right)^{x+1}. \]

Evaluate the infinite sum \( f(1) + f(2) + f(3) + f(4) + \ldots \)

2. Find the area of the shape bounded by the following relations:

\[ y \leq |x| - 2. \]
\[ y \geq |x| - 4. \]
\[ y \leq 0. \]

\(|x|\) denotes the absolute value of \(x\).

3. An equilateral triangle with side length 6 is inscribed inside a circle. What is the diameter of the largest circle that can fit in the circle but outside of the triangle?

4. Given \( \sin x - \tan x = \sin x \tan x \), solve for \(x\) in the interval \((0, 2\pi)\), exclusive.

5. How many rectangles are there in a 6 by 6 square grid?

6. Find the lateral surface area of a cone with radius 3 and height 4.

7. From 9 positive integer scores on a 10-point quiz, the mean is 8, the median is 8, and the mode is 7. Determine the maximum number of perfect scores possible on this test.

8. If \(i = \sqrt{-1}\), evaluate the following continued fraction:

\[ 2i + \frac{1}{2i + \frac{1}{2i + \ldots}}. \]

9. The cubic polynomial \(x^3 - px^2 + px - 6\) has roots \(p\), \(q\), and \(r\). What is \((1-p)(1-q)(1-r)\)?

10. (Variant on a Classic.) Gilnor is a merchant from Cutlass, a town where 10% of the merchants are thieves. The police utilize a lie detector that is 90% accurate to see if Gilnor is one of the thieves. According to the device, Gilnor is a thief. What is the probability that he really is one?