

Johns Hopkins Mathematics Tournament

April 23, 2005

TEAM QUESTION PAPER

1. Consider the following function:

$$f(x) = \left(\frac{1}{2}\right)^x - \left(\frac{1}{2}\right)^{x+1}.$$

Evaluate the infinite sum $f(1) + f(2) + f(3) + f(4) + \dots$

2. Find the area of the shape bounded by the following relations:

$$y \leq |x| - 2.$$

$$y \geq |x| - 4.$$

$$y \leq 0.$$

$|x|$ denotes the absolute value of x .

3. An equilateral triangle with side length 6 is inscribed inside a circle. What is the diameter of the largest circle that can fit in the circle but outside of the triangle?
4. Given $\sin x - \tan x = \sin x \tan x$, solve for x in the interval $(0, 2\pi)$, exclusive.
5. How many rectangles are there in a 6 by 6 square grid?
6. Find the lateral surface area of a cone with radius 3 and height 4.
7. From 9 positive integer scores on a 10-point quiz, the mean is 8, the median is 8, and the mode is 7. Determine the maximum number of perfect scores possible on this test.
8. If $i = \sqrt{-1}$, evaluate the following continued fraction:

$$2i + \frac{1}{2i + \frac{1}{2i + \frac{1}{2i + \dots}}}$$

9. The cubic polynomial $x^3 - px^2 + px - 6$ has roots p , q , and r . What is $(1-p)(1-q)(1-r)$?
10. (*Variant on a Classic.*) Gilnor is a merchant from Cutlass, a town where 10% of the merchants are thieves. The police utilize a lie detector that is 90% accurate to see if Gilnor is one of the thieves. According to the device, Gilnor is a thief. What is the probability that he really is one?