Johns Hopkins Mathematics Tournament

April 23, 2005

Combinatorics Question Paper

1. How many 3-digit whole numbers are divisible by 1, 2, 3, 4, and 5?

2. An aquarium contains 6 male goldfish and 10 female goldfish. If two fish are taken out at random, what is the chance that they will be of opposite gender?

3. Find N such that any N distinct positive integers chosen from [1,25] inclusive contain both an even and an odd number.

4. A positive integer N has seven factors, the fourth greatest of which is 8. Find N.

5. Three 6-sided dice are rolled. If no two of the resulting numbers are the same, what is the probability that one of them is a 4?

6. Two different numbers are taken from the the set $\{0,1,2,3,4,5,6,7,8,9,10\}$. Determine the probability that their sum and positive difference are both multiples of 4.

7. Three people each flip two fair coins. Compute the probability that exactly two of the people flipped one head and one tails.

8. A three-man jury has two members who make the correct decision with probability p and a third member who flips a fair coin to decide (majority rules). Determine the probability of the jury making the correct decision in a given case.

9. You are given one chance to play the following game. You roll three fair dice; you win \$2 if you roll exactly one 6, \$4 if you roll exactly two 6s, and \$6 if you roll all three 6s. How much money do you expect to win?

10. A box contains some quarters and some dimes, with probability q of drawing a quarter. I remove half the quarters and an equal number of dimes, then double the number of dimes, adding an equal number of quarters. The chance of drawing a dime is now q. Find q.