

Johns Hopkins Mathematics Tournament

April 23, 2005

ALGEBRA QUESTION PAPER

1. If a and b are real numbers, determine the minimum value of:

$$a^2 + b^2 - 6a.$$

2. A rectangle's perimeter and area both have a value of 25. What is the length of its longer side?

3. If $b > 1$, what is the positive difference between the two roots of the following equation?

$$x^2 + 2bx + 1 = 0.$$

4. Consider the following function:

$$f(x) = 2^x - x - 4.$$

Compute $f(0) + f(1) + f(2) + \dots + f(8)$.

5. Determine the sum of the positive factors of 496.

6. Determine $a + b + c + d$ if:

$$3a + 2b - c - d = 1.$$

$$2a + 2b - c + 2d = 2.$$

$$4a - 2b - 3c + d = 3.$$

$$8a + b - 6c + d = 4.$$

7. An equilateral triangle with area $\sqrt{3}$ and located in the first quadrant has one vertex located on the y -axis and one vertex located on the origin. Find the coordinates of the third vertex.

8. a , b , and c are distinct positive integers satisfying:

$$(a - b + c)(b - c + a)(c - a + b) = 15.$$

Find abc .

9. How many integer pairs (a, b) fulfill the following condition?

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{2}.$$

10. Evaluate the infinite sum:

$$\frac{3}{(1 \cdot 2)^2} + \frac{5}{(2 \cdot 3)^2} + \frac{7}{(3 \cdot 4)^2} + \frac{9}{(4 \cdot 5)^2} + \dots$$