1. In a Super Smash Brothers tournament, $\frac{1}{2}$ of the contestants play as Fox, $\frac{1}{3}$ of the contestants play as Falco, and $\frac{1}{6}$ of the contestants play as Peach. Given that there were 40 more people who played either Fox or Falco than who played Peach, how many contestants attended the tournament?

   Answer: 60

   Solution: Let x denote the number of contestants in the tournament. Then $\frac{1}{2}x + \frac{1}{3}x - \frac{1}{6}x = 40$. Thus, $\frac{2}{3}x = 40$ and hence $x = \boxed{60}$ contestants attended the tournament.

2. Compute the number of ways 6 girls and 5 boys can line up if all 11 people are distinguishable and no two girls stand next to each other.

   Answer: 86400

   Solution: Note that the lineup must be GBGBGBGBGBG. There are $6! \cdot 5! = \boxed{86400}$ ways that they can line up.

3. The line $y = x + 2015$ intersects the parabola $y = x^2$ at two points, (a, b) and (c, d). Compute $a + c$.

   Answer: 1

   Solution: Note that a and c satisfy the equation $x^2 - x - 2015 = 0$. By the quadratic formula, the two solutions are $\frac{1+\sqrt{(-1)^2-4(1)(-2015)}}{2(1)}$ and $\frac{1-\sqrt{(-1)^2-4(1)(-2015)}}{2(1)}$. Therefore, the two solutions sum to be $\frac{1}{2} + \frac{1}{2} = \boxed{1}$.

4. Initially 2 miles apart, two cars are driving north on a straight freeway. The southern car is driving 80 mph and the northern car has a speed of 50 mph. A very fast bird, initially sitting on the front car flies off directly at the other car at a speed of 90 mph! When the bird approaches a car it instantly turns around and flies the other direction. What is the total distance that the bird flies, in miles, before getting smashed between the cars?

   Answer: 6

   Solution: The bird flies continuously while the cars are apart so if the cars crash in time T then the bird will have flown $90T$ miles. The distance between the cars decreases at the difference of their speeds so they will crash in $T = \frac{2mi}{80-50mi/h} = 4min$. Thus the bird will travel $\boxed{6}$ miles!

5. A certain high school has exactly 1000 lockers, numbered from 1 to 1000, all initially closed. Mark and Matt decide to practice lockpicking after school one day. Mark first opens every locker whose number has exactly 3 factors, starting with locker 4. Matt then opens every locker whose number is a power of 2, starting with locker 1. If Matt encounters a locker that Mark has already opened, he closes it and reopens it for extra practice. Compute the number of lockers that will be open when both Mark and Matt finish.

   Answer: 20

   Solution: Numbers with exactly three factors must be squares of primes (so the factors are 1, p, and p^2). Between 1 and 1000 there are 11 such numbers: 2^2, 3^2, 5^2, 7^2, 11^2, 13^2, 17^2, 19^2, 23^2, 29^2, 31^2. Furthermore, there are 10 powers of 2 between 1 and 1000: $2^0, 2^1, \ldots 2^9$. The number 4 is in each list, so there are a total of $\boxed{20}$ distinct lockers that Mark and Matt will open.

6. An integer n is *almost square* if there exists a perfect square k^2 such that $|n - k^2| = 1$ and k is a positive integer. How many positive integers less than or equal to 2015 are almost square?

Answer: 87

Solution: First, there are $\lfloor\sqrt{2015}\rfloor = 44$ perfect squares less than 2015. For each perfect square $1, 4, 9, \ldots$ there are 2 almost square integers corresponding to the perfect square. However, we've included 0, so there are $44 \cdot 2 - 1 = \boxed{87}$ positive almost square integers less than or equal to 2015.

7. In a right triangle, dropping an altitude to the hypotenuse divides the hypotenuse into two segments of length 2 and 3 respectively. What is the area of the triangle?

Answer: $\mathbf{\frac{5\sqrt{6}}{2}}$

Solution: Denote the right triangle ABC with hypotenuse BC. Let D be the intersection of the altitude and BC and let $CD = 2$ and $BD = 3$. Triangle ACD is similar to triangle ABC so $\frac{AC}{CD} = \frac{BC}{AC}$. Thus, $AC = \sqrt{BC \cdot CD} = \sqrt{5 \cdot 2} = \sqrt{10}$. Triangle ABD is similar to triangle ABC so $\frac{AB}{BD} = \frac{BC}{AB}$. Thus, $AB = \sqrt{BC \cdot BD} = \sqrt{5 \cdot 3} = \sqrt{15}$. Therefore, the area of ABC is $\frac{1}{2} \cdot \sqrt{10} \cdot \sqrt{15} = \boxed{\dfrac{5\sqrt{6}}{2}}$.

8. Let $f(x) = ax^2 + bx + c$ where $a \neq 0$. Find d where $0 < d < 1$ such that $f(0) = 2014$, $f(d^2) = 2015$, $f(d) = 2016$, and the sum of the roots of f equals 0.

Answer: $\mathbf{\frac{1}{\sqrt{2}}}$

Solution: The sum of the roots of a quadratic is $-\frac{b}{2a}$, so the sum of the roots of f equals 0 if and only if $b = 0$. Now, the three values of f give us

$$\begin{aligned} c &= 2014 \\ ad^4 + bd^2 + c &= 2015 \\ ad^2 + bd + c &= 2016 \end{aligned}$$

Subtracting the first equation from the second two and substituting $b = 0$, we obtain

$$\begin{aligned} ad^4 &= 1 \\ ad^2 &= 2 \end{aligned}$$

Dividing equation one by equation two, we get that

$$d^2 = \frac{1}{2}$$

Therefore, it follows that $d = \pm\frac{1}{\sqrt{2}}$. Since we want $0 < d < 1$, we conclude that $\boxed{d = \dfrac{1}{\sqrt{2}}}$.

9. There are four seats arranged in a circle and a person is sitting on one of the seats. He rolls a die 6 times. For each roll of the die, if it lands on 4, he moves one seat clockwise. Otherwise, he moves k seats counterclockwise where k is the number he rolled. Compute the probability that he ends up on the same seat he originally started on.

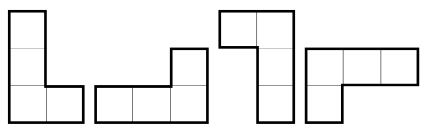
Answer: $\mathbf{\frac{61}{243}}$

Solution: First, note that for each roll of the die, he has an equal probability of moving to any of the remaining seats. Let p_n be the probability that he returns to his seat after n rolls. Then

$p_{n+1} = \frac{1}{3}(1 - p_n)$ and $p_1 = 0$. Therefore, we compute:

$$p_1 = 0$$
$$p_2 = \frac{1}{3}$$
$$p_3 = \frac{2}{9}$$
$$p_4 = \frac{7}{27}$$
$$p_5 = \frac{20}{81}$$
$$p_6 = \boxed{\frac{61}{243}}$$

10. How many ways are there to tile a 4×14 grid using the following tiles?



Answer: 85

Solution: Let a_{2n} denote the number of ways to tile a $4 \times 2n$ grid. The leftmost part of any tiling must either consist of a 4×2 block formed from the 2nd and 4th tiles or a 4×4 block formed from either from two of each of the 1st and 3rd tiles or from a pinwheel-like design formed from one of each of the four tiles. Thus, we see that $a_{2n} = a_{2n-2} + 2a_{2n-4}$. Since $a_2 = 1$ and $a_4 = 3$, we have the sequence $\{1, 3, 5, 11, 21, 43, 85...\}$. It follows that a_{2n} is the nth number in this sequence and hence $a_{14} = \boxed{85}$.