

Time limit: 60 minutes.

Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.

No calculators.

1. In a gathering of one-eyed, four-legged cyclops and 210 humans, there are 672 legs present. Determine how many eyes there are.
2. A triangle with side lengths 6, 10, and 14 has area x . A triangle with side lengths 9, 15, and 21 has area y . Compute $\frac{x}{y}$.
3. Lynnelle really loves peanut butter, but unfortunately she cannot afford to buy her own. Her roommate Jane also likes peanut butter, and Jane buys a new 100mL jar every month. Lynnelle has decided to steal some peanut butter from Jane's jar every day immediately after Jane eats, but to make sure Jane doesn't notice Lynnelle never steals more than 20mL and never steals so much that the amount remaining in the jar is more than halved. For example, if 50mL of peanut butter remains in the jar then Lynnelle will steal 20mL that day (since half of 50mL is 25mL, and Lynnelle will steal at most 20mL in one day), and if 8mL remains then Lynnelle will steal 4mL that day (leaving 4mL, half of 8mL). If Jane eats a constant 10mL of peanut butter each day (or the rest of the jar, if the jar has less than 10mL in it) until the jar is empty, compute the amount Lynnelle steals (in mL).
4. Consider a unit square $ABCD$. Let E be the midpoint of BC and F the intersection of AC and DE . Compute the area of triangle ADF .
5. Tyrant Tal, a super genius, wants to create an army of Tals. He and his clones can clone themselves, but the process takes an entire hour. Once the clone is created, it must wait 2 hours before creating its own clones. So at the end of the first hour, there could be 2 Tals (the original and 1 clone), and a clone created during the fifth hour can clone itself during the eighth hour. Compute the maximum size of his army after 10 hours (Tyrant Tal starts by himself and is a part of his own army).
6. How many ways are there to draw a path between all dots in a 3×3 grid exactly once, where a path from dot to dot may only go horizontally or vertically?
7. Find the unique $x > 0$ such that $\sqrt{x} + \sqrt{x + \sqrt{x}} = 1$.
8. Andy has two identical cups, the first one is full of water and the second one is empty. He pours half the water from the first cup into the second, then a third of the water in the second into the first, then a fourth of the water from the first into the second and so on. Compute the fraction of the water is in the first cup right before the 2015th transfer.
9. Consider a unit cube and a plane that slices through it. The plane passes through the midpoints of two adjacent edges on the top face, two on the bottom face, and the center of the cube. Compute the area of the cross section.
10. Let the sequence a be defined as $a_0 = 2, a_n = 1 + a_0 \cdot a_1 \cdot \dots \cdot a_{n-1}$. Calculate $\sum_{i=0}^{2015} \frac{1}{a_i}$. Give your answer in terms of a_{2016} .