Time limit: 60 minutes.

Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.

No calculators.

1. Let $f(x) = x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$. Compute $f'(2)$.

2. There are 10 contestants in the Stanford Mountaineering Tournament, numbered 1 to 10. At time t the height of the contestant number n is given by $x_n(t) = t^n$. Compute the average speed of the 10 contestants at time $t = 2$.

3. Moor is trying to paint the interval $[0, 5]$ using red and green paints. For some reason, painting at the point x using red paint costs $2x$ dollars per unit length and using green paint costs x^2 dollars per length. What is the minimum amount of money Moor needs to spend to paint the entire interval if he's allowed to change colors as he paints?

4. Compute
$$\frac{d}{dx} \prod_{n=1}^{2014} \left(x + \frac{1}{n} \right) \Bigg|_{x=0}$$

5. For some positive pairs of real numbers (α, β) the following limit exists and is nonzero. Compute it in terms of α and β:
$$\lim_{x \to 0} \frac{\sin x^\alpha}{\cos x^\beta - 1}$$

6. Compute
$$\int_0^2 \sqrt{(2 - x) \left(\sqrt{x} + \sqrt{x + 2} \right)^2} \, dx.$$

7. Given that it converges, compute the following infinite product:
$$\prod_{n=1}^{\infty} \frac{5^{2^{-n}} + 3^{2^{-n}}}{2}.$$

8. Compute
$$\frac{1}{\pi} \int_0^\pi \left(\frac{\sin(10x)}{\sin x} \right)^2 dx$$
.

9. Compute:
$$\sum_{k=1}^{\infty} \frac{(\pi - 3)^{2^{k-1}}}{1 - (\pi - 3)^{2^k}}.$$

10. Consider the real-valued differential equation $u''(x) = u^2(x) - u^5(x)$. Suppose that $u'(0) = 7$ and $u(0) = 2$, compute the largest value of $|u'(x)|$.