

HOMEWORK 4

Due: 9 October 2009 in recitation hour.

(1) Let $d$ be a pseudometric defined on a set $M$. Namely, $d : M \times M \to [0, \infty)$ satisfying

- $d(x, x) = 0$ for all $x \in M$,
- $d(x, y) = d(y, x)$ for all $x, y \in M$,
- $d(x, y) \leq d(x, z) + d(z, y)$ for all $x, y, z \in M$.

Open balls, Cauchy sequences and convergent sequences can be defined in terms of the quasi-metric $d$ as in the case of usual metric.

Let $(a_n)$ be a convergent sequence in this quasi-metric space. Is the limit unique? Prove or give a counterexample.

(2) Let $M = [0, \infty)$. Show that

$$d(x, y) = \begin{cases} 
|\arctan(x) - \arctan(y)| & \text{if } x \neq 0 \text{ and } y \neq 0; \\
|\arctan(x) - \frac{\pi}{2}| & \text{if } x \neq 0 \text{ and } y = 0; \\
0 & \text{if } x = y = 0.
\end{cases}$$

defines a metric on $M$.

Does the sequence $(a_n) = n$ converges?

(3) From the book Chapter 3 Problems 18, 23, 27, 31, 37, 42,

Suggested Exercises Do not return these

Chapter 3 Problems 20, 21, 22, 24, 25, 26, 27