

Formula: Inverse of a
2x2 matrix.

Consider

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \text{and}$$

$$B = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Flip on diagonal,
negative ~~inverse~~ of f
diagonal

Compute

$$AB = \begin{bmatrix} ad - cb & 0 \\ 0 & -bc + da \end{bmatrix}.$$

$$= \begin{bmatrix} ad - cb & 0 \\ 0 & ad - cb \end{bmatrix}.$$

$$= (ad - bc) I.$$

If $ad - bc \neq 0$ then

$$A \underbrace{\begin{pmatrix} 1 & \\ ad - bc & B \end{pmatrix}}_{A^{-1}} = I$$

$$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

if $ad-bc \neq 0$.

If $ad-bc = 0$. Then

$AB = 0$ in other words

$$A \begin{bmatrix} d \\ -c \end{bmatrix} = 0 \quad \text{and}$$

$$A \begin{bmatrix} -b \\ a \end{bmatrix} = 0.$$

So either $A = 0$ matrix

or A has a non-zero
vector in $\text{Ker}(A)$

$\Rightarrow A$ is not invertible.

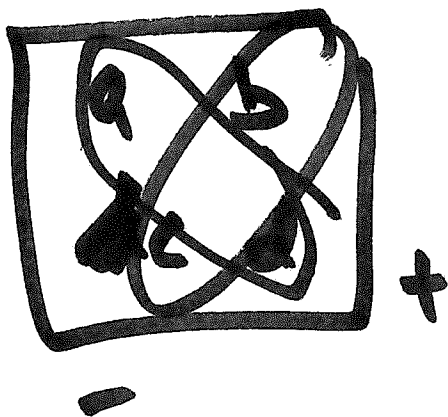
Fact: A 2×2 matrix A
is invertible if and only
if

$$ad - bc \neq 0,$$

in which case

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

The quantity
 $ad - bc$



is called the determinant.

It's non-zero when A is
invertible.

★ The determinant determines
if A is invertible.

Examples:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}.$$

$$\begin{aligned} \det(A) &= 1 \cdot (-1) - 1 \cdot 1 \\ &= -2. \end{aligned}$$

$-2 \neq 0$ so A is invertible.

$$\begin{aligned} A^{-1} &= \frac{1}{-2} \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{bmatrix}. \end{aligned}$$

Example

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$$

$$\det(A) = 1 \cdot 2 - 2 \cdot 1 = 0.$$

So A is not invertible.

We could also see that

A is not invertible

2nd row is a multiple of
the 1st i.e. $\text{Rref}(A) \neq I$.