Name: ______________________________

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MATH 201, QUIZ 5.

Because the vectors $v_1 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ form a basis for $\mathbf{R}^2$, every vector $x \in \mathbf{R}^2$ can be written as

$$x = c_1 v_1 + c_2 v_2$$

for some unique pair of coefficients $c_1, c_2 \in \mathbf{R}^2$.

(1) Sketch the subspaces $L_1 = \text{span}(v_1)$ and $L_2 = \text{span}(v_2)$ on a single coordinate plane. Choose any vector v **not contained** in this pair of subspaces (really, choose any random one) and draw it on your coordinate plane. Draw the (approximate) locations of the unique vectors $c_1 v_1$ and $c_2 v_2$ so that $v = c_1 v_1 + c_2 v_2$. (no computation is necessary)

(2) Let $A := \begin{bmatrix} 1/3 & -1/3 \\ -2/3 & 2/3 \end{bmatrix}$. Find a matrix A' so that if

$$x = c_1 v_1 + c_2 v_2,$$

then $\begin{bmatrix} c_1' \\ c_2' \end{bmatrix} := A' \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$ is the unique coordinate vector needed to express Ax as

$$Ax = c_1' v_1 + c_2' v_2.$$

Find an invertible matrix S such that $SA'S^{-1} = A$.

(3) Let v be your vector from part (1). Label the (approximate) location of Av on your coordinate plane (no computation is necessary). (Hint: the transformation A is sometimes called the projection onto L_1 along L_2. Think about how this geometric description is related your expression A' for the matrix A in (v_1, v_2)-coordinates.)

Do not write in this space.