Quiz 4 Solution:

First, we find A^{-1} for $A = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 3 & 0 & 0 & 5 \end{bmatrix}$

by row reducing

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 1 & 0 \\ 3 & 0 & 0 & 5 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \updownarrow \mathrm{II}, \mathrm{III} \\ \\ \mathrm{IV} - 3\mathrm{I} \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & -3 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \mathrm{I} + 2\mathrm{IV} \\ \mathrm{II} + \mathrm{III} \\ \\ \cdot -1 \end{array} \longrightarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -5 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 & 0 & 0 & -1 \end{array}\right]$$

and we conclude that $\boxed{A^{-1} = \begin{bmatrix} -5 & 0 & 0 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & -1 \end{bmatrix}}$

Now, to solve $A\vec{x} = \vec{b}$, we multiply both sides by A^{-1}, so we have a <u>unique solution</u>, namely $\vec{x} = A^{-1}\vec{b}$.

Here, $\vec{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ and $A^{-1}\vec{b} = \begin{bmatrix} -3 \\ 0 \\ 0 \\ 2 \end{bmatrix}$ so the <u>unique</u> $\underline{\vec{x}}$ such that $\vec{x}$ solves $A\vec{x} = \vec{b}$ is $\boxed{\vec{x} = \begin{bmatrix} -3 \\ 0 \\ 0 \\ 2 \end{bmatrix}}$.