5. Let \( B = \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix} \)

(a) (10 points) Find the singular value decomposition for \( B \).
(b) (10 points) Find all eigenvalues of $B$ and their associated eigenspaces. Is $B$ diagonalizable?
(c) (6 points) Do the following:

1. Draw a pair of coordinate planes below. Label them domain of $B$ and range of $B$, respectively.

2. On the coordinate plane labeled domain of $B$, sketch the unit circle. Sketch each eigenspace of $B$. Label all unit length eigenvectors for $B$.

3. On the coordinate plane labeled range of $B$, sketch the ellipse obtained by applying $B$ to all vectors in the unit circle. Label the principal axes of this ellipse. Sketch each eigenspace of $B$. Determine the points of intersection between the image of the unit circle under $B$ and each eigenspace explicitly. What are the lengths of these eigenvectors for $B$ which lie on the image of the unit circle?
(d) (3 points) Verify that each (real) eigenvalue $\lambda$ of $B$ satisfies $\sigma_2 \leq |\lambda| \leq \sigma_1$. Can you explain this inequality using your illustration in part (c)? Would you expect this inequality to hold for the (real) eigenvalues of a general $2 \times 2$ matrix?