Problem 1. Consider the system of linear equations:

\begin{align*}
x_1 + 2x_2 + x_3 &= 1, \\
5x_1 + 6x_2 + 5x_3 &= 16, \\
x_1 + 2x_2 + 3x_3 &= 1.
\end{align*}

(1) Express this system as a matrix equation of the form \(Ax = b\).
(2) Is \(A\) invertible? If so, compute \(A^{-1}\).
(3) Determine the set of all solutions for the system above.

Problem 2. (1) Let \(L\) be the line of slope 2 in \(\mathbb{R}^2\) through the origin. Find a matrix \(A\) such that the orthogonal projection onto \(L\) (i.e. \(x \mapsto x^\parallel\)) is given by \(x \mapsto Ax\).
(2) Compute \(A^2\) and verify \(A^2 = A\). Can you explain this relation geometrically?

Problem 3. Let \(B = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}\).

(1) Find bases for the image of \(B\) and the kernel of \(B\).
(2) State the rank-nullity theorem and verify that it holds for \(B\).
(3) Is \(\begin{bmatrix} 3 \\ 7 \\ 11 \end{bmatrix}, \begin{bmatrix} 4 \\ 8 \\ 12 \end{bmatrix}\) (the third and fourth column of \(B\)) a basis for the image of \(B\)? Explain your answer.

Problem 4. (Conceptual Problem) Let \(B\) be the matrix in problem 3. Assume that \(C_1\) and \(C_2\) are matrices such that the product \(C_1BC_2\) is defined. What values are possible for the rank of \(C_1BC_2\)? Explain your answer.