(Practice) Midterm 2

(1) Consider the basis $\beta = \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right)$ for $\mathbf{R}^2$. Let $R : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation defined by rotation by $\pi/4$-radians (45 degrees) counterclockwise about the origin.

  (a) Find the matrix $[R]_\beta$ describing R in β-coordinates.
  (b) Determine $[R]_\beta^{2019}$.

(2) (a) Let $V = \text{span}\left(\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \right)$. Find a basis for $V^\perp$.

  (b) Find an orthogonal matrix of the form:

$$A = \frac{1}{2} \begin{bmatrix} 1 & 1 & a_1 & a_2 \\ 1 & -1 & b_1 & b_2 \\ 1 & 1 & c_1 & c_2 \\ 1 & -1 & d_1 & d_2 \end{bmatrix}.$$

  (c) Calculate the determinant $\text{Det}(A)$ of your matrix from part (b).

(3) Fit a linear function $f(x) = ax + b$ to the data set $(0, 5), (1, 9), (2, 16)$ using least squares. Sketch your solution. In what sense does your solution best fit this data set?

(4) Is the matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ diagonalizable? Justify your answer.