(Practice) Midterm 2

(1) Consider the basis \( \beta = \left( \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) \) for \( \mathbb{R}^2 \). Let \( R : \mathbb{R}^2 \to \mathbb{R}^2 \) be the linear transformation defined by rotation by \( \pi/4 \)-radians (45 degrees) counterclockwise about the origin.

(a) Find the matrix \([R]_\beta\) describing \( R \) in \( \beta \)-coordinates.
(b) Determine \([R]_{\beta}^{2019}\).

(2) (a) Let \( V = \text{span} \left( \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \right) \). Find a basis for \( V^\perp \).

(b) Find an orthogonal matrix of the form:

\[
A = \frac{1}{2} \begin{bmatrix}
1 & 1 & a_1 & a_2 \\
1 & -1 & b_1 & b_2 \\
1 & 1 & c_1 & c_2 \\
1 & -1 & d_1 & d_2
\end{bmatrix}.
\]

(c) Calculate the determinant \( \text{Det}(A) \) of your matrix from part (b).

(3) Fit a linear function \( f(x) = ax + b \) to the data set \((0, 5), (1, 9), (2, 16)\) using least squares. Sketch your solution. In what sense does your solution best fit this data set?

(4) Is the matrix \( A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix} \) diagonalizable? Justify your answer.