Lecture I: Solving Systems of Linear Equations (Part I).

Warm-up Problem:

Graph the solution set to

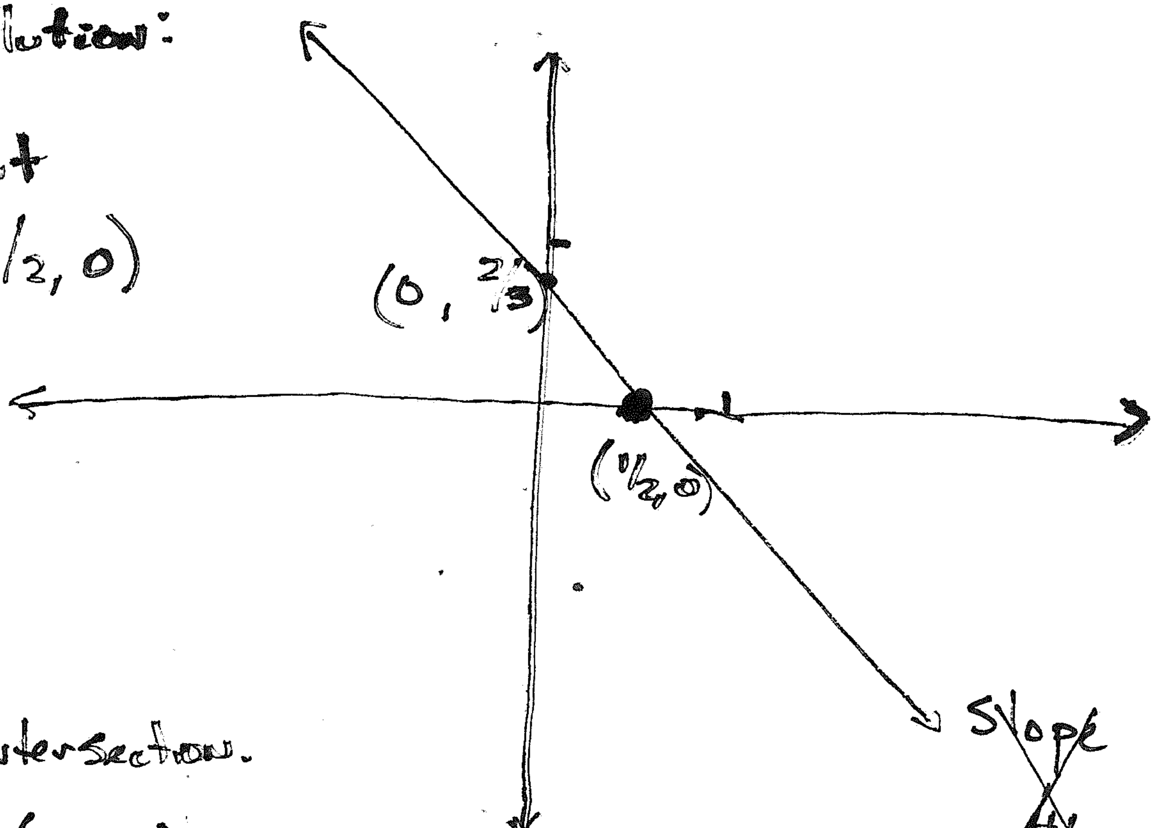
$$4x + 3y = 2$$

in the (x,y)-plane.

Solution:

x int

$(1/2, 0)$



y intersection.

$(0, 2/3)$

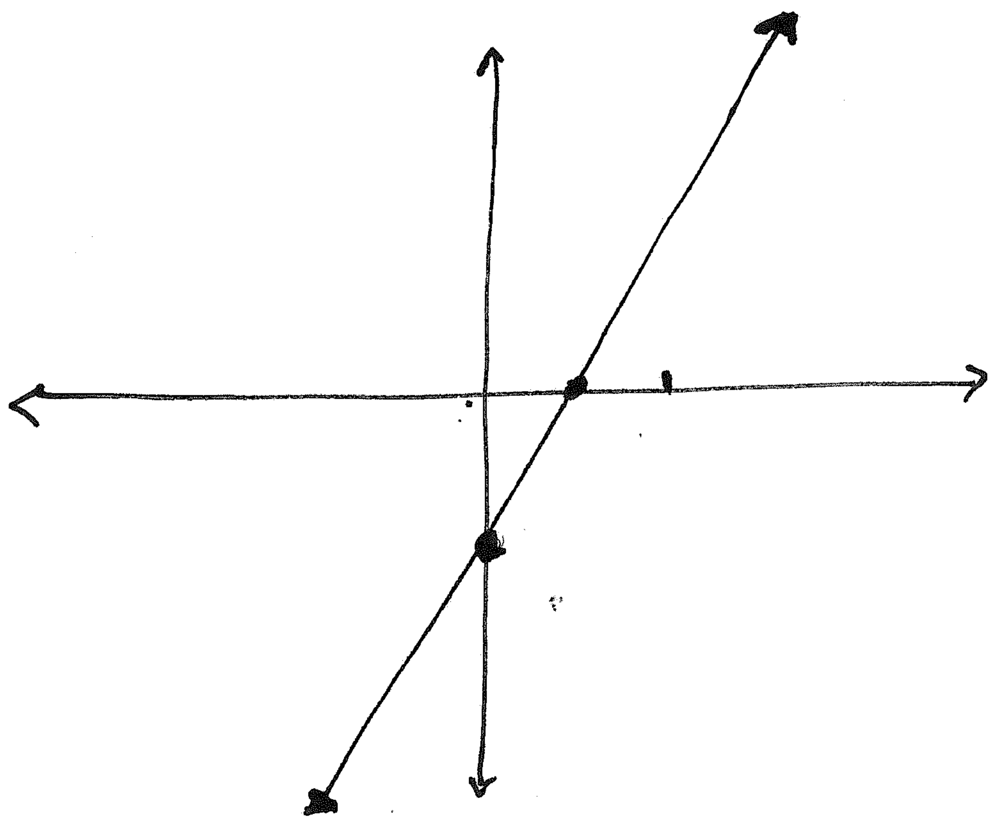
~~Slope~~

$\cancel{-4/3}$.

Doesn't help us graph.

Warm-up II: Graph the solution set to

$$2x - y = 1$$



x int $\left(\frac{1}{2}, 0\right)$

y int $(0, -1)$

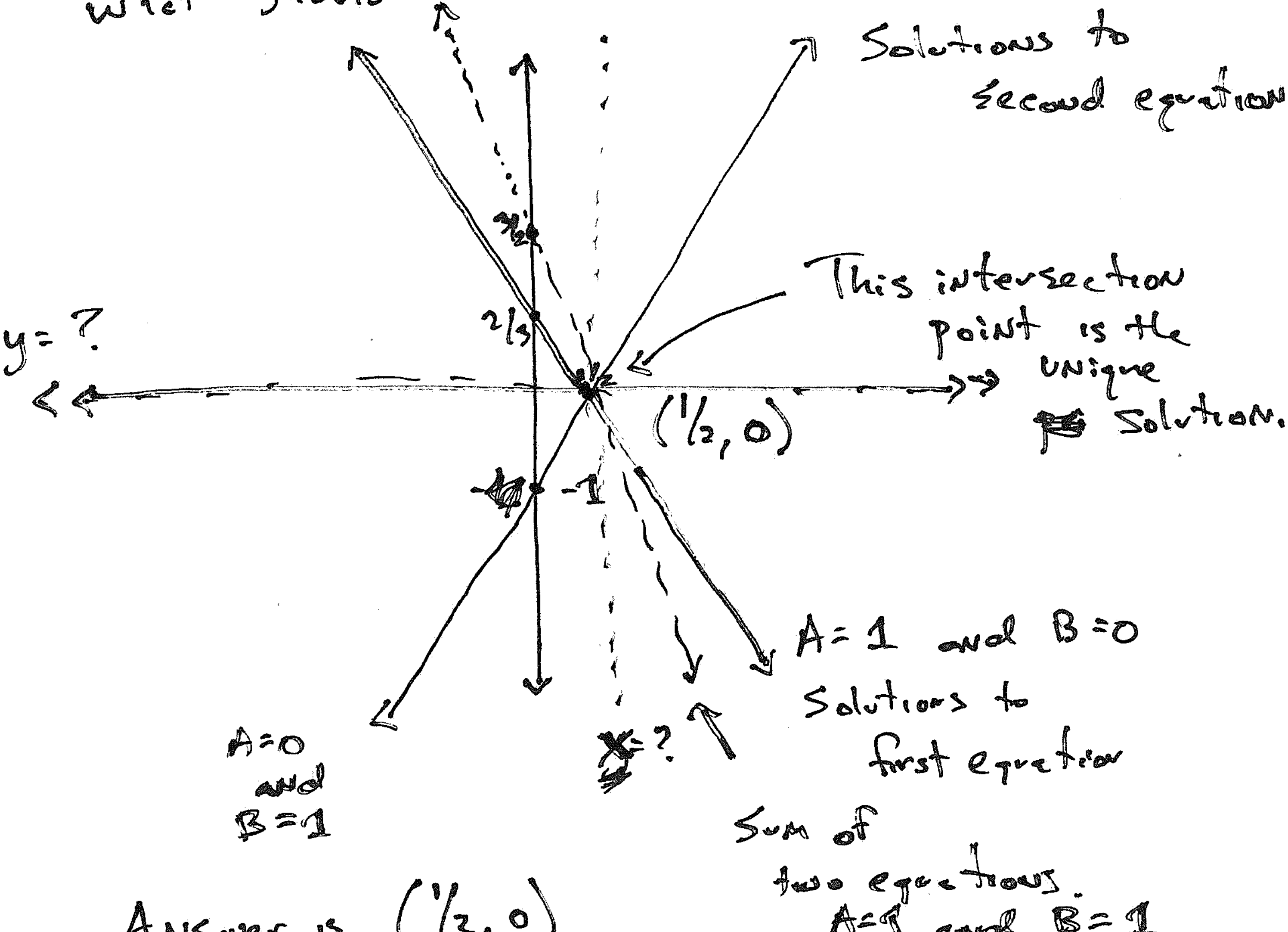
Problem I: Find all (x, y) such that

$$4x + 3y = 2$$

and

$$2x - y = 1.$$

What should this solution set be?



Answer is $(1/2, 0)$.

More generally you'd expect a unique solution when you have the equations for two lines.

* We can understand the solution set geometrically which gives us insight into "how many" solutions there ~~sol~~ should be.

* To find actual solutions we will make algebraic manipulations.

Lets pretend we didn't know the solution already.

Solving the system of linear equations would mean giving expressions of the form

vert. $x =$ a number
hor. $y =$ a number.

geometrically these are also equations of lines.

Find other lines which pass through the same solution set.

Algebraically

$$(4x+3y)+(2x-y) = 2+1$$

New line that goes though our solution point.

$$6x+2y=3.$$

More generally, for any scalars A and B

$$A(4x+3y) + B(2x-y) = 2A+B.$$

is a line through the solution.

When we solve the problem algebraically we are looking for the A and B which gives us the vert. (resp. horizontal line).

Algebraic Solution:

$$\begin{aligned} 4x + 3y &= 2 \\ 2x - y &= 1 \end{aligned} \xRightarrow{\mathrm{I} - 2 \cdot \mathrm{II}} \begin{aligned} (4x + 3y) - 2(2x - y) &= 2 - 2 \cdot 1 \\ 2x - y &= 1 \end{aligned}$$

$$\Rightarrow \begin{aligned} 5y &= 0 \\ 2x - y &= 1 \end{aligned}$$

divide by 5 in I

$$\Rightarrow \begin{aligned} y &= 0 \\ 2x - y &= 1 \end{aligned}$$

I:
II + I

$$\Rightarrow \begin{aligned} y &= 0 \\ (2x - y) + y &= 1 + 0 \end{aligned}$$

$$\Rightarrow \begin{aligned} y &= 0 \\ 2x &= 1 \end{aligned}$$

$$\Rightarrow \boxed{\begin{aligned} y &= 0 \\ x &= 1/2. \end{aligned}}$$

Let's analyze how we solved problem I.

Goal: Replace our skew lines with nicer lines (vert + horizontal)

Idea behind solution: Algorithmically eliminate how many ~~non-zero~~ variables occurred in each equation

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Problem II: Solve

$$\begin{aligned} x + 2y + 3z &= 39 \\ x + 2y + 2z &= 34 \\ 3x + 2y + z &= 26. \end{aligned}$$

The solutions to each eq. individually is a plane in $\mathbb{R}^3$.

Solution: Expect that there should be a unique solution. (Since 3 planes usually intersect in a point)

want (ideally)

$x = ?$
$y = ?$
$z = ?$

Next step: solve this algebraically.

Step 1: eliminate x-terms from equations 2 and 3.

$$x + 2y + 3z = 39$$
$$(x + 2y + 2z) - (x + 2y + 3z) = 34 - 39$$
$$(3x + 2y + z) - 3(x + 2y + 3z) = 26 - 3 \cdot 39.$$

$$x + 2y + 3z = 39$$
$$\square \qquad - z = -5$$
$$\square \quad -4y - 8z = (26 - 1[illegible])$$

we have to switch the order of these equations since there is no y-term in the second

$$x + 2y + 3z = 39$$
$$-4y - 8z = \cancel{-22}91\cancel{\ }$$
$$-z = -5$$

divide by leading coeff.

$$x + 2y + 3z = 39$$
$$y + 2z = -91/4$$
$$z = 5.$$

... Continued Next time.