Let $A := \begin{bmatrix} \frac{9}{13} & \frac{2}{13} \\ -\frac{17}{13} & \frac{15}{13} \end{bmatrix}$.

(1) Find a scaling and rotation matrix $R := \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ and an invertible matrix $S$ such that $A = SRS^{-1}$. Using technology find the approximate angle of rotation and describe $R$ geometrically.

(2) Find the singular value decomposition of $S$.

(3) Determine the image of the circle of radius $k$ under $S$. Give your solution in the form:

$$E_k := \{k\cos(\theta)(Av_1) + k\sin(\theta)(Av_2) : \theta \in \mathbf{R}\}.$$

(4) Sketch $E_k$. Draw and label the principal axes, label the intercepts of $E$ with the principal axes, and give the formula for $E_k$ in the coordinate system defined by the principal axes. How is your drawing related to the singular value decomposition of $S$?

(5) Find the value of $k$ such that $e_1 \in E_k$. Using technology, find the approximate angle $\theta$ such that $e_1 = k\cos(\theta)(Av_1) + k\sin(\theta)(Av_2)$ in your parametrization of $E_k$.

(6) Compute $e_1, Ae_1, A^2e_1, A^3e_1, A^4e_1$ using technology. These points lie on ellipse $E_k$. Explain this, and sketch these points on the ellipse $E_k$ (for the appropriate value of $k$).

(7) Find the integer $n < 100$ such that $|A^n e_1|$ is maximized. Do not use technology (beyond your use of technology above). Explain your answer.