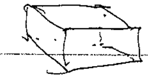
Notes from MO.113,
Mon. 29 Aug 2011

## Some History



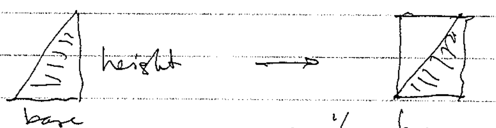
≈ 3300 BC

The origins of mathematics is obscure. Mud bricks date back to 7000 BC [Mergarh in Baluchistan], so the Indus Valley civilization, which made these things in bulk, presumably knew that

Volume of a rectangular solid = height·width·depth

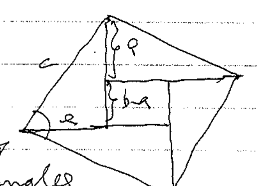
The Sumerians had advanced calendrics, which passed them on to the Babylonians. That's why the circle has 360 degrees (dating back to a prehistoric calendar; the Chinese used 365 degrees, but they came later).

Clearly the area of a right triangle




$= \frac{1}{2}\cdot \text{base}\cdot\text{height}$.

Pythagoras's theorem




$$c^2 = 4\cdot\tfrac{1}{2}ab + (b-a)^2$$
$$= 2ab + b^2 - 2ab + a^2$$
$$= a^2 + b^2$$

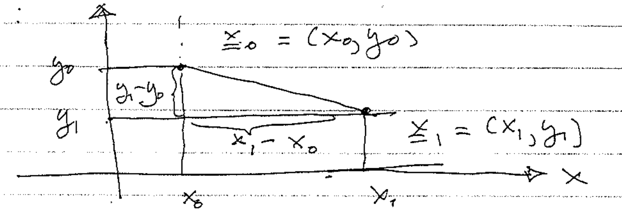
complementary angles

⇒ This is a square of area $c^2$

dates back to 1000 BC or so.

ii)

Much of the understanding of geometry was lost during the dark ages, but around 1500 AD (cf. eg. Descartes) we begin to see things like this:

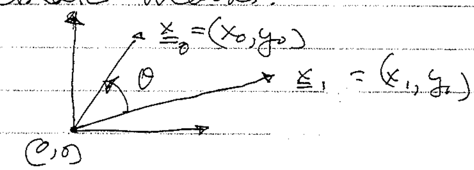



⇒ distance between $\underline{x}_0$ and $\underline{x}_1$

$$= \sqrt{(x_1-x_0)^2 + (y_1-y_0)^2}$$

which means we can calculate distances purely by algebraic means.

Similarly




$$\boxed{\cos\theta = \frac{x_0 x_1 + y_0 y_1}{\sqrt{x_1^2+y_1^2}\cdot\sqrt{x_0^2+y_0^2}}}$$

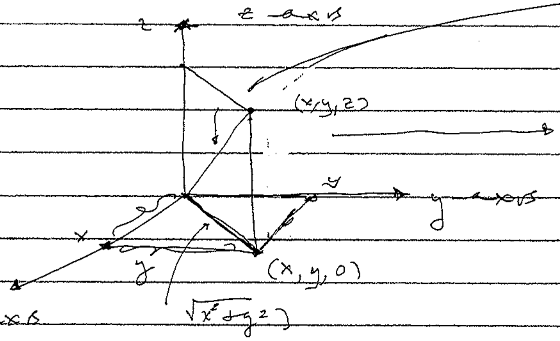
allows us to calculate angles algebraically.

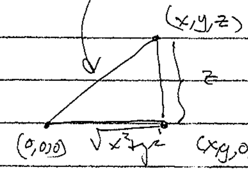
Do you know this formula?

iii)

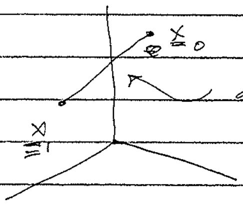
$$\text{dist} = \sqrt{\left(\sqrt{x^2+y^2}\right)^2 + z^2} = \sqrt{x^2+y^2+z^2}$$

This generalizes as follows:




so, we




$\text{dist}^2$ from $\underline{x}_0 = (x_0, y_0, z_0)$

to $\underline{x}_1 = (x_1, y_1, z_1)$

$= (x_0 - x_1)^2 + (y_0 - y_1)^2 + (z_0 - z_1)^2$

Which leads to the following DEFINITION

LaTex: \mathbb R

$$\mathbb{R}^n = \{ (x_1, \dots, x_n) \mid x_k \in \mathbb{R} \}$$

$=$ n-dimensional Euclidean space, $n = 1, 2, \dots$

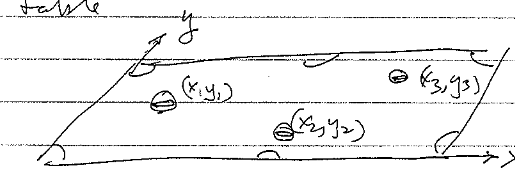
is the set of ordered n-tuples of real numbers

LaTex: \bf x

An element $\underline{x} \in \mathbb{R}^n$ is called a point, and $x_k$ is its k-th coordinate.

iv)

Ex A configuration of n billiard balls on a pool table



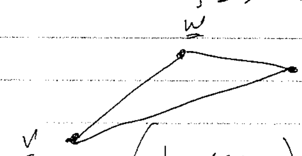

defines a point $(x_1, y_1; x_2, y_2; x_3, y_3, \dots) \in \mathbb{R}^{2n}$

Definition The distance

$$|\underline{v} - \underline{w}| = \left[ \sum_{k=1}^{k=n} (v_k - w_k)^2 \right]^{1/2}$$

between two points $\underline{v}, \underline{w} \in \mathbb{R}^n$ satisfies

- $\text{dist}(\underline{v}, \underline{w}) = |\underline{v} - \underline{w}| = 0 \iff \underline{v} = \underline{w}$
- $\text{dist}(\underline{v}, \underline{w}) = \text{dist}(\underline{w}, \underline{v})$
- ☆ $\text{dist}(\underline{v}, \underline{w}) + \text{dist}(\underline{w}, \underline{u}) \geq \text{dist}(\underline{v}, \underline{u})$




(In case you're worried) → ie a straight line is the shortest distance between two points, $\boxed{\text{in } n \text{ dimensions}}$

v)

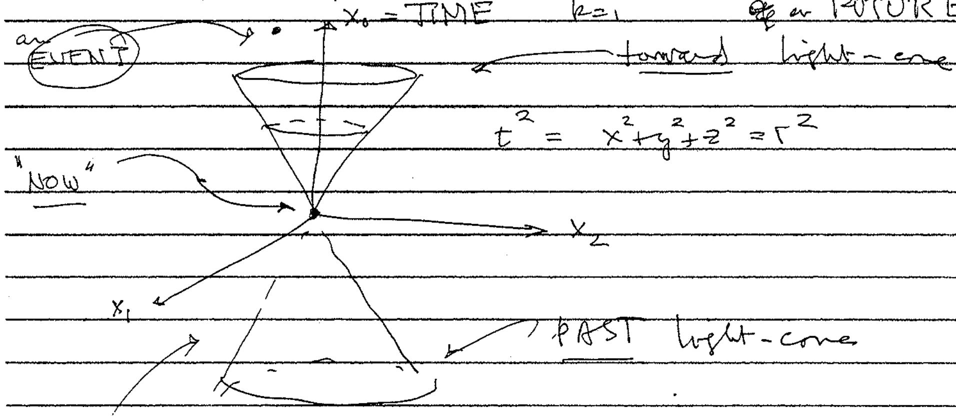
Extra Credit Digression: The 20th Century

~ 1915

Einstein & Minkowski had the bright idea of considering a 4-dimensional space-time $\mathbb{R}^4$ in which

$\boxed{\text{it's not really a distance!}}$

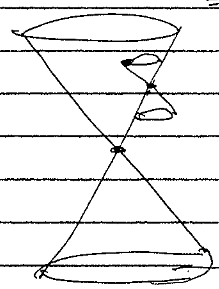
$$dist_M^2 = (v_0 - w_0)^2 - \sum_{k=1}^{k=3} (v_k - w_k)^2$$




$t^2 = x^2 + y^2 + z^2 = r^2$

surface of the cone = EVENTS we can see NOW = which are simultaneous from our point of view (Relativistic)

IMPORTANT FACT



SIMULTANEITY is NOT an EQUIVALENCE RELATION

WEDS:

MORE about equivalence relations

(& their relation to SET THEORY!)