SPECIAL TRIGONOMETRIC SUBSTITUTIONS

This is the subject of section 8.4.

- If you encounter the expression $\sqrt{a^2 - x^2}$, use the substitution $x = a \sin u$, and then:

$$\sqrt{a^2 - x^2} = a \cos u$$
$$dx = a \cos u du$$

- If you encounter the expression $\sqrt{a^2 + x^2}$, use the substitution $x = a \tan u$, and then:

$$\sqrt{a^2 + x^2} = a \sec u$$
$$dx = a \sec^2 u du$$

- If you encounter the expression $\sqrt{x^2 - a^2}$, use the substitution $x = a \sec u$, and then:

$$\sqrt{x^2 - a^2} = a \tan u$$
$$dx = a \tan u \sec u du$$

These set of substitutions would reduce some of the ugly indefinite integrals involving square root expressions like $\sqrt{\pm 1 \pm x^2}$ or $\frac{1}{\sqrt{\pm 1 \pm x^2}}$ (or even $\frac{1}{(1-x^2)^{3/2}}$, as in Example 1 of 8.4) to friendly indefinite integrals involving (powers of) trigonometric functions (which we know how to solve from 8.3.