TRIG IDENTITIES WITH $\sin$ AND $\cos$

- $\sin^2 + \cos^2 = 1$
- $\sin^2 x = \frac{1-\cos(2x)}{2}, \cos^2 x = \frac{1+\cos(2x)}{2}$
- $\sin(2x) = 2\sin x \cos x$

Comments:

- The second identity is extremely useful when dealing with even powers of $\sin$ and $\cos$, as in $\int \sin^4 x dx \cos^2 x$ for example, since it lowers the degree of complexity by doubling the angle (the expression is converted into a simple expression involving $\sin(2x)$ and $\cos(2x)$).
- Also, because of the first relation, every time we use the substitution $x = \sin u$ for example, we can determine all other trig. functions of u immediately: $\cos u = \sqrt{1-x^2}$, $\tan u = \frac{x}{\sqrt{1-x^2}}$, $\sec u = \frac{1}{\sqrt{1-x^2}}$, etc.. (This is particularly useful if we use a substitution ($x = \sin u$ for example) and at the end we want to give the final answer in terms of x and not of u.
- Keep in mind that every time you make a substitution, dx gets transformed as well. Example: if $x = \sin u$, then $dx = \cos u du$.