CONVERGENCE TESTS FOR POSITIVE SERIES

The following convergence tests can be applied to positive series:

**Integral test**
If \( f(x) \geq 0 \) is a decreasing continuous function, then
\[
\sum_{n=1}^{\infty} f(n) \sim \int_{1}^{\infty} f(x) \, dx
\]

**Comparison test**
\( a_n \)

a) If \( 0 \leq a_n \leq b_n \) and \( \sum b_n \) is convergent then \( \sum a_n \) is convergent.

b) If \( 0 \leq a_n \leq b_n \) and \( \sum a_n \) is divergent, then \( \sum b_n \) is divergent.

c) If \( \lim_{n \to \infty} \frac{a_n}{b_n} = L \neq 0 \), then \( \sum a_n \sim \sum b_n \).

**Ratio test**
Let \( \lambda = \lim_{n \to \infty} \frac{a_{n+1}}{a_n} \) (assuming the limit exists).

- If \( \lambda < 1 \) then \( \sum a_n \) is convergent.
- If \( \lambda > 1 \) then \( \sum a_n \) is divergent.
- If \( \lambda = 1 \), the test is inconclusive.

**Root test**
Let \( \rho := \lim_{n \to \infty} a_n^{1/n} \) (assuming the limit exists).

- If \( \rho < 1 \) then \( \sum a_n \) is convergent.
- If \( \rho > 1 \) then \( \sum a_n \) is divergent.
- If \( \rho = 1 \), the test is inconclusive.