CONVERGENCE TESTS FOR POSITIVE SERIES

The following convergence tests can be applied to positive series:

Integral test
If $f(x) \geq 0$ is a decreasing continuous function, then

$$\sum_{n=1}^{\infty} f(n) \sim \int_{1}^{\infty} f(x)dx$$

Comparison test
a) **If** $0 \leq a_n \leq b_n$ and $\sum b_n$ is convergent **then** $\sum a_n$ is convergent.
b) **If** $0 \leq a_n \leq b_n$ and $\sum a_n$ is divergent, **then** $\sum b_n$ is divergent.
c) **If** $\lim_{n\to\infty} \frac{a_n}{b_n} = L \neq 0$, **then** $\sum a_n \sim \sum b_n$.

Ratio test Let $\lambda = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$ (assuming the limit exists).

- **If** $\lambda < 1$ **then** $\sum a_n$ is convergent.
- **If** $\lambda > 1$ **then** $\sum a_n$ is divergent.
- **If** $\lambda = 1$, the test is inconclusive.

Root test Let $\rho := \lim_{n\to\infty} a_n^{1/n}$ (assuming the limit exists).

- **If** $\rho < 1$ **then** $\sum a_n$ is convergent.
- **If** $\rho > 1$ **then** $\sum a_n$ is divergent.
- **If** $\rho = 1$, the test is inconclusive.