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The synthetic theory of ∞ -categories vs the synthetic theory of ∞ -categories

joint with Dominic Verity and Michael Shulman



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The motivation for ∞ -categories



Mere **1-categories** are insufficient habitats for sophisticated mathematical objects like motives that have higher-dimensional transformations encoding relevant “higher homotopical information.”

A better setting is given by **∞ -categories**, where the usual **sets** of morphisms are enriched to **spaces** of morphisms.

\leadsto Thus, we want to extend 1-category theory (e.g., adjunctions, limits and colimits, universal properties, Kan extensions) to ∞ -category theory.

First problem: it is hard to say exactly what an ∞ -category is.

The idea of an ∞ -category



∞ -categories are the nickname that Lurie gave to $(\infty, 1)$ -categories, which are categories **weakly enriched** over homotopy types.

The schematic idea is that an ∞ -category should have

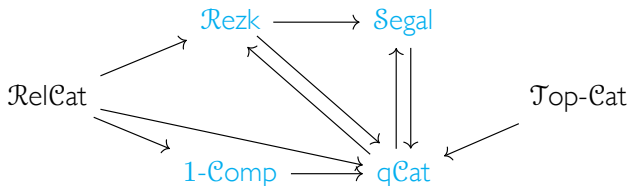
- objects
- 1-arrows between these objects
- with composites of these 1-arrows witnessed by invertible 2-arrows
- with composition associative up to invertible 3-arrows (and unital)
- with these witnesses coherent up to invertible arrows all the way up

But this definition is tricky to make precise.

Models of ∞ -categories



The notion of ∞ -category is made precise by several models:



- topological categories and relative categories are the simplest to define but do not have enough maps between them
- $\left\{ \begin{array}{l} \text{quasi-categories (nee. weak Kan complexes),} \\ \text{Rezk spaces (nee. complete Segal spaces),} \\ \text{Segal categories, and} \\ \text{(saturated 1-trivial weak) 1-complicial sets} \end{array} \right.$
each have enough maps and also an internal hom, and in fact any of these categories can be enriched over any of the others

The analytic vs synthetic theory of ∞ -categories



Q: How might you develop the category theory of ∞ -categories?

Strategies:

- work **analytically** to give categorical definitions and prove theorems using the combinatorics of one model

(eg., Joyal, Lurie, Gepner-Haugseng, Cisinski in **qCat**;
Kazhdan-Varshavsky, Rasekh in **Rezk**; Simpson in **Segal**)

- work **synthetically** to give categorical definitions and prove theorems in all four models **qCat**, **Rezk**, **Segal**, **1-Comp** at once

(R-Verity: an **∞ -cosmos** axiomatizes the common features of the categories **qCat**, **Rezk**, **Segal**, **1-Comp** of ∞ -categories)

- work **synthetically** in a simplicial type theory augmenting HoTT to prove theorems in **Rezk**

(R-Shulman: an **∞ -category** is a type with unique binary composites in which isomorphism is equivalent to identity)



0. The analytic theory of ∞ -categories

“ ∞ -category theory for experts”

1. The synthetic theory of ∞ -categories (in an ∞ -cosmos)

“ ∞ -category theory for graduate students”

2. The synthetic theory of ∞ -categories (in homotopy type theory)

“ ∞ -category theory for undergraduates”



The synthetic theory of ∞ -categories (in an ∞ -cosmos)

∞ -cosmoi of ∞ -categories



An ∞ -cosmos is an axiomatization of the properties of \mathbf{qCat} .

defn. An ∞ -cosmos has:

- ∞ -categories A, B as objects
- functors between ∞ -categories $f: A \rightarrow B$, which define the points of a quasi-category $\mathbf{Fun}(A, B) = B^A$
- a class of isofibrations $E \twoheadrightarrow B$ with familiar closure properties
- so that (flexible weighted simplicially enriched) limits of diagrams of ∞ -categories and isofibrations exist.

Theorem. \mathbf{qCat} , \mathbf{Rezk} , \mathbf{Segal} , and $\mathbf{1-Comp}$ define ∞ -cosmoi.

Henceforth ∞ -category and ∞ -functor are technical terms that mean the objects and morphisms of some ∞ -cosmos.

The homotopy 2-category



The **homotopy 2-category** of an ∞ -cosmos is a strict 2-category whose:

- objects are the ∞ -categories A, B in the ∞ -cosmos
- 1-cells are the ∞ -functors $f: A \rightarrow B$ in the ∞ -cosmos
- 2-cells we call ∞ -natural transformations $A \begin{array}{c} \xrightarrow{f} \\ \Downarrow \gamma \\ \xrightarrow{g} \end{array} B$ which are defined to be homotopy classes of 1-simplices in $\text{Fun}(A, B)$

Prop. **Equivalences** in the homotopy 2-category

$$\begin{array}{ccc} A & \begin{array}{c} \xrightarrow{f} \\ \Downarrow \gamma \\ \xrightarrow{g} \end{array} & B \\ A & \begin{array}{c} \xrightarrow{\text{id}_A} \\ \Downarrow \cong \\ \xrightarrow{gf} \end{array} & A \\ B & \begin{array}{c} \xrightarrow{\text{id}_B} \\ \Downarrow \cong \\ \xrightarrow{fg} \end{array} & B \end{array}$$

coincide with **equivalences** in the ∞ -cosmos.

Thus, non-evil 2-categorical definitions are “homotopically correct.”

Adjunctions between ∞ -categories



defn. An **adjunction** between ∞ -categories is an adjunction in the homotopy 2-category, consisting of:

- ∞ -categories A and B
- ∞ -functors $u: A \rightarrow B, f: B \rightarrow A$

- ∞ -natural transformations $B \begin{array}{c} \xrightarrow{\text{id}_B} \\ \Downarrow \eta \\ \xrightarrow{uf} \end{array} B$ and $A \begin{array}{c} \xrightarrow{fu} \\ \Downarrow \epsilon \\ \xrightarrow{\text{id}_A} \end{array} A$

satisfying the **triangle equalities**

$$\begin{array}{c}
 B \xlongequal{\quad} B \\
 \begin{array}{c} u \nearrow \\ \Downarrow \epsilon \\ f \searrow \end{array} \Downarrow \eta \begin{array}{c} \nearrow u \\ \Downarrow \epsilon \\ \searrow f \end{array} \\
 A \xlongequal{\quad} A
 \end{array}
 =
 \begin{array}{c}
 B \\
 \begin{array}{c} \uparrow \\ (=) \\ \downarrow \end{array} \\
 A
 \end{array}
 \qquad
 \begin{array}{c}
 B \xlongequal{\quad} B \\
 \begin{array}{c} \searrow f \\ \Downarrow \eta \\ \nearrow u \end{array} \Downarrow \epsilon \begin{array}{c} \searrow f \\ \Downarrow \epsilon \\ \nearrow u \end{array} \\
 A \xlongequal{\quad} A
 \end{array}
 =
 \begin{array}{c}
 B \\
 \begin{array}{c} \downarrow \\ (=) \\ \uparrow \end{array} \\
 A
 \end{array}$$

Write $f \dashv u$ to indicate that f is the **left adjoint** and u is the **right adjoint**.

The 2-category theory of adjunctions



Since an adjunction between ∞ -categories is just an adjunction in the homotopy 2-category, all 2-categorical theorems about adjunctions become theorems about adjunctions between ∞ -categories.

Prop. Adjunctions compose:

$$C \begin{array}{c} \xrightarrow{f'} \\ \perp \\ \xleftarrow{u'} \end{array} B \begin{array}{c} \xrightarrow{f} \\ \perp \\ \xleftarrow{u} \end{array} A \quad \rightsquigarrow \quad C \begin{array}{c} \xrightarrow{ff'} \\ \perp \\ \xleftarrow{u'u} \end{array} A$$

Prop. Adjoints to a given functor $u: A \rightarrow B$ are unique up to canonical isomorphism: if $f \dashv u$ and $f' \dashv u$ then $f \cong f'$.

Prop. Any equivalence can be promoted to an adjoint equivalence: if $u: A \xrightarrow{\sim} B$ then u is left and right adjoint to its equivalence inverse.

Composing adjunctions



Prop. Adjunctions compose:

$$\begin{array}{ccc}
 C & \begin{array}{c} \xrightarrow{f'} \\ \perp \\ \xleftarrow{u'} \end{array} & B & \begin{array}{c} \xrightarrow{f} \\ \perp \\ \xleftarrow{u} \end{array} & A & \rightsquigarrow & C & \begin{array}{c} \xrightarrow{ff'} \\ \perp \\ \xleftarrow{u'u} \end{array} & A
 \end{array}$$

Proof: The composite 2-cells

$$\begin{array}{ccc}
 C & \xlongequal{\quad} & C & & C & & C \\
 \downarrow f' & & \downarrow \eta' & & \downarrow \epsilon' & & \downarrow f' \\
 B & \xlongequal{\quad} & B & & B & \xlongequal{\quad} & B \\
 \downarrow f & & \downarrow \eta & & \downarrow \epsilon & & \downarrow f \\
 A & & A & & A & \xlongequal{\quad} & A \\
 & & \uparrow u & & \uparrow u & & \uparrow u
 \end{array}$$

define the unit and counit of $ff' \dashv u'u$ satisfying the triangle equalities.

Initial and terminal elements in an ∞ -category



defn. An ∞ -category A has a terminal element iff $1 \begin{array}{c} \xleftarrow{!} \\ \perp \\ \xrightarrow{t} \end{array} A$.

Prop. Right adjoints preserve terminal elements.

Proof: Compose the adjunctions $1 \begin{array}{c} \xleftarrow{!} \\ \perp \\ \xrightarrow{t} \end{array} A \begin{array}{c} \xleftarrow{f} \\ \perp \\ \xrightarrow{u} \end{array} B$.

More generally:

Prop. Right adjoints preserve limits and left adjoints preserve colimits.

Proof: The usual one!

The universal property of adjunctions



defn. Any ∞ -category A has an ∞ -category of arrows A^2 , pulling back

to define the comma ∞ -category:

$$\begin{array}{ccc}
 \mathrm{Hom}_A(f, g) & \longrightarrow & A^2 \\
 \downarrow (\mathrm{cod}, \mathrm{dom}) & \lrcorner & \downarrow (\mathrm{cod}, \mathrm{dom}) \\
 C \times B & \xrightarrow{g \times f} & A \times A
 \end{array}$$

Prop. $A \begin{array}{c} \xleftarrow{f} \\ \perp \\ \xrightarrow{u} \end{array} B$ if and only if $\mathrm{Hom}_A(f, A) \simeq_{A \times B} \mathrm{Hom}_B(B, u)$.

Prop. If $f \dashv u$ with unit η and counit ϵ then

- η is initial in $\mathrm{Hom}_B(B, u)$ over B .
- ϵ is terminal in $\mathrm{Hom}_A(f, A)$ over A .



2

The synthetic theory of ∞ -categories
(in homotopy type theory)

The Curry-Howard-Voevodsky correspondence



type theory	set theory	logic	homotopy theory
A	set	proposition	space
$x : A$	element	proof	point
$\emptyset, 1$	$\emptyset, \{\emptyset\}$	\perp, \top	$\emptyset, *$
$A \times B$	set of pairs	A and B	product space
$A + B$	disjoint union	A or B	coproduct
$A \rightarrow B$	set of functions	A implies B	function space
$x : A \vdash B(x)$	family of sets	predicate	fibration
$x : A \vdash b : B(x)$	fam. of elements	conditional proof	section
$\prod_{x:A} B(x)$	product	$\forall x.B(x)$	space of sections
$\sum_{x:A} B(x)$	disjoint sum	$\exists x.B(x)$	total space
$p : x =_A y$	$x = y$	proof of equality	path from x to y
$\sum_{x,y:A} x =_A y$	diagonal	equality relation	path space for A



The identity type family is freely generated by the terms $\text{refl}_x : x =_A x$.

Path induction. If $B(x, y, p)$ is a type family dependent on $x, y : A$ and $p : x =_A y$, then to prove $B(x, y, p)$ it suffices to assume y is x and p is refl_x . I.e., there is a function

$$\text{path-ind} : \left(\prod_{x:A} B(x, x, \text{refl}_x) \right) \rightarrow \left(\prod_{x,y:A} \prod_{p:x=_A y} B(x, y, p) \right).$$

The intended model



$$\begin{array}{ccccccc} \text{Set}^{\Delta^{\text{op}} \times \Delta^{\text{op}}} & \supset & \mathcal{R}\text{eedy} & \supset & \text{Segal} & \supset & \mathcal{R}\text{ezk} \\ \parallel & & \parallel & & \parallel & & \parallel \\ \text{bisimplicial sets} & & \text{types} & & \text{types with} & & \text{types with} \\ & & & & \text{composition} & & \text{composition} \\ & & & & & & \text{\& univalence} \end{array}$$

Theorem (Shulman). Homotopy type theory is modeled by the category of **Reedy fibrant** bisimplicial sets.

Theorem (Rezk). ∞ -categories are modeled by **Rezk spaces** aka complete Segal spaces.

Shapes in the theory of the directed interval



Our types may depend on other types and also on **shapes** $\Phi \subset 2^n$, polytopes embedded in a directed cube, defined in a language

$$\top, \perp, \wedge, \vee, \equiv \quad \text{and} \quad 0, 1, \leq$$

satisfying **intuitionistic logic** and **strict interval** axioms.

$$\Delta^n := \{(t_1, \dots, t_n) : 2^n \mid t_n \leq \dots \leq t_1\} \quad \text{e.g.} \quad \Delta^1 := 2$$

$$\Delta^2 := \left\{ \begin{array}{ccc} & (t,t) & (1,1) \\ & \diagdown & | \\ (0,0) & & (1,t) \\ & \diagup & \\ & (t,0) & (1,0) \end{array} \right.$$

$$\partial\Delta^2 := \{(t_1, t_2) : 2^2 \mid (t_2 \leq t_1) \wedge ((0 = t_2) \vee (t_2 = t_1) \vee (t_1 = 1))\}$$

$$\Lambda_1^2 := \{(t_1, t_2) : 2^2 \mid (t_2 \leq t_1) \wedge ((0 = t_2) \vee (t_1 = 1))\}$$

Extension types



Formation rule for extension types

$$\frac{\Phi \subset \Psi \text{ shape} \quad A \text{ type} \quad a : \Phi \rightarrow A}{\left\langle \begin{array}{ccc} \Phi & \xrightarrow{a} & A \\ \downarrow & \searrow \text{dashed} & \\ \Psi & & \end{array} \right\rangle \text{ type}}$$

A term $f : \left\langle \begin{array}{ccc} \Phi & \xrightarrow{a} & A \\ \downarrow & \searrow \text{dashed} & \\ \Psi & & \end{array} \right\rangle$ defines

$$f : \Psi \rightarrow A \text{ so that } f(t) \equiv a(t) \text{ for } t : \Phi.$$

The simplicial type theory allows us to *prove* equivalences between extension types along composites or products of shape inclusions.

Hom types



The **hom type** for A depends on two terms in A :

$$x, y : A \vdash \text{Hom}_A(x, y)$$

$$\text{Hom}_A(x, y) := \left\langle \begin{array}{ccc} \partial\Delta^1 & \xrightarrow{[x,y]} & A \\ \Downarrow & \nearrow & \\ \Delta^1 & & \end{array} \right\rangle \text{ type}$$

A term $f : \text{Hom}_A(x, y)$ defines an **arrow** in A from x to y .

Semantically, $\sum_{x,y:A} \text{Hom}_A(x, y)$ recovers the ∞ -category of arrows A^2 in the ∞ -cosmos $\mathcal{R}ezk$ and $\text{Hom}_A(x, y)$ recovers the **comma ∞ -category** from x to y .

Segal types \equiv types with binary composition



A type A is *Segal* iff every composable pair of arrows has a unique composite, i.e., for every $f : \text{Hom}_A(x, y)$ and $g : \text{Hom}_A(y, z)$ the type

$$\left\langle \begin{array}{ccc} \Lambda_1^2 & \xrightarrow{[f,g]} & A \\ \Downarrow & \searrow \text{dashed} & \\ \Delta^2 & & \end{array} \right\rangle \text{ is contractible.}$$

Semantically, a Reedy fibrant bisimplicial set A is *Segal* if and only if $A^{\Delta^2} \rightarrow A^{\Lambda_1^2}$ has contractible fibers.

By contractibility, $\left\langle \begin{array}{ccc} \Lambda_1^2 & \xrightarrow{[f,g]} & A \\ \Downarrow & \searrow \text{dashed} & \\ \Delta^2 & & \end{array} \right\rangle$ has a unique inhabitant. Write

$g \circ f : \text{Hom}_A(x, z)$ for its inner face, the composite of f and g .

Identity arrows



For any $x : A$, the constant function defines a term

$$\text{id}_x := \lambda t. x : \text{Hom}_A(x, x) := \left\langle \begin{array}{ccc} \partial\Delta^1 & \xrightarrow{[x, x]} & A \\ \Downarrow & \nearrow & \\ \Delta^1 & & \end{array} \right\rangle,$$

which we denote by id_x and call the **identity arrow**.

For any $f : \text{Hom}_A(x, y)$ in a Segal type A , the term

$$\lambda(s, t). f(t) : \left\langle \begin{array}{ccc} \Lambda_1^2 & \xrightarrow{[\text{id}_x, f]} & A \\ \Downarrow & \nearrow & \\ \Delta^2 & & \end{array} \right\rangle$$

witnesses the unit axiom $f = f \circ \text{id}_x$.

Associativity of composition

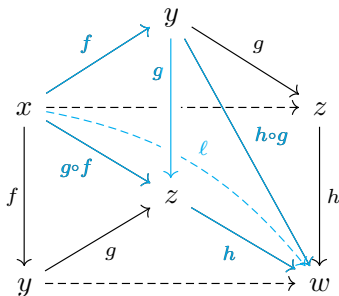


Let A be a Segal type with arrows

$$f : \text{Hom}_A(x, y), \quad g : \text{Hom}_A(y, z), \quad h : \text{Hom}_A(z, w).$$

Prop. $h \circ (g \circ f) = (h \circ g) \circ f.$

Proof: Consider the composable arrows in the Segal type $\Delta^1 \rightarrow A$:



Composing defines a term in the type $\Delta^2 \rightarrow (\Delta^1 \rightarrow A)$ which yields a term $l : \text{Hom}_A(x, w)$ so that $l = h \circ (g \circ f)$ and $l = (h \circ g) \circ f.$

Isomorphisms



An arrow $f: \text{Hom}_A(x, y)$ in a Segal type is an **isomorphism** if it has a two-sided inverse $g: \text{Hom}_A(y, x)$. However, the type

$$\sum_{g: \text{Hom}_A(y, x)} (g \circ f = \text{id}_x) \times (f \circ g = \text{id}_y)$$

has higher-dimensional structure and is *not* a **proposition**. Instead define

$$\text{iso}(f) := \left(\sum_{g: \text{Hom}_A(y, x)} g \circ f = \text{id}_x \right) \times \left(\sum_{h: \text{Hom}_A(y, x)} f \circ h = \text{id}_y \right).$$

For $x, y : A$, the **type of isomorphisms** from x to y is:

$$x \cong_A y := \sum_{f: \text{Hom}_A(x, y)} \text{iso}(f).$$

Rezk types $\equiv \infty$ -categories



By path induction, to define a map

$$\text{path-to-iso} : (x =_A y) \rightarrow (x \cong_A y)$$

for all $x, y : A$ it suffices to define

$$\text{path-to-iso}(\text{refl}_x) := \text{id}_x.$$

A Segal type A is Rezk iff every isomorphism is an identity, i.e., iff the map

$$\text{path-to-iso} : \prod_{x, y : A} (x =_A y) \rightarrow (x \cong_A y)$$

is an equivalence.

Discrete types $\equiv \infty$ -groupoids



Similarly by path induction define

$$\text{path-to-arr}: (x =_A y) \rightarrow \text{Hom}_A(x, y)$$

for all $x, y : A$ by $\text{path-to-arr}(\text{refl}_x) := \text{id}_x$.

A type A is **discrete** iff every arrow is an identity, i.e., iff **path-to-arr** is an equivalence.

Prop. A type is discrete if and only if it is Rezk and all of its arrows are isomorphisms.

Proof:

$$\begin{array}{ccc} x =_A y & \xrightarrow{\text{path-to-arr}} & \text{Hom}_A(x, y) \\ & \searrow \text{path-to-iso} & \nearrow \\ & x \cong_A y & \end{array}$$

∞ -categories for undergraduates



defn. An ∞ -groupoid is a type in which arrows are equivalent to identities:

path-to-arr: $(x =_A y) \rightarrow \text{Hom}_A(x, y)$ is an equivalence.

defn. An ∞ -category is a type

- which has unique binary composites of arrows:

$$\left\langle \begin{array}{ccc} \Lambda_1^2 & \xrightarrow{[f,g]} & A \\ \Downarrow & & \nearrow \\ \Delta^2 & & \end{array} \right\rangle \quad \text{is contractible}$$

- and in which isomorphisms are equivalent to identities:

path-to-iso: $(x =_A y) \rightarrow (x \cong_A y)$ is an equivalence.

Covariant type families \equiv categorical fibrations



A type family $x : A \vdash B(x)$ over a Segal type A is **covariant** if for every $f : \text{Hom}_A(x, y)$ and $u : B(x)$ there is a unique lift of f with domain u .

The codomain of the unique lift defines a term $f_* u : B(y)$.

Prop. For $u : B(x)$, $f : \text{Hom}_A(x, y)$, and $g : \text{Hom}_A(y, z)$,

$$g_*(f_* u) = (g \circ f)_* u \quad \text{and} \quad (\text{id}_x)_* u = u.$$

Prop. If $x : A \vdash B(x)$ is covariant then for each $x : A$ the fiber $B(x)$ is discrete. Thus covariant type families are fibered in ∞ -groupoids.

Prop. Fix $a : A$. The type family $x : A \vdash \text{Hom}_A(a, x)$ is covariant.

The Yoneda lemma



Let $x : A \vdash B(x)$ be a covariant family over a Segal type and fix $a : A$.

Yoneda lemma. The maps

$$\text{ev-id} := \lambda\phi.\phi(a, \text{id}_a) : \left(\prod_{x:A} \text{Hom}_A(a, x) \rightarrow B(x) \right) \rightarrow B(a)$$

and

$$\text{yon} := \lambda u.\lambda x.\lambda f.f_* u : B(a) \rightarrow \left(\prod_{x:A} \text{Hom}_A(a, x) \rightarrow B(x) \right)$$

are inverse equivalences.

Corollary. A natural isomorphism $\phi : \prod_{x:A} \text{Hom}_A(a, x) \cong \text{Hom}_A(b, x)$ induces an identity $\text{ev-id}(\phi) : b =_A a$ if the type A is Rezk.

The dependent Yoneda lemma



Yoneda lemma. If A is a Segal type and $B(x)$ is a covariant family dependent on $x : A$, then evaluation at (a, id_a) defines an equivalence

$$\text{ev-id} : \left(\prod_{x:A} \text{Hom}_A(a, x) \rightarrow B(x) \right) \rightarrow B(a)$$

The Yoneda lemma is a “directed” version of the “transport” operation for identity types, suggesting a dependently-typed generalization analogous to the full induction principle for identity types.

Dependent Yoneda lemma. If A is a Segal type and $B(x, y, f)$ is a covariant family dependent on $x, y : A$ and $f : \text{Hom}_A(x, y)$, then evaluation at (x, x, id_x) defines an equivalence

$$\text{ev-id} : \left(\prod_{x,y:A} \prod_{f:\text{Hom}_A(x,y)} B(x, y, f) \right) \rightarrow \prod_{x:A} B(x, x, \text{id}_x)$$

Dependent Yoneda is directed path induction



Slogan: the dependent Yoneda lemma is directed path induction.

Path induction. If $B(x, y, p)$ is a type family dependent on $x, y : A$ and $p : x =_A y$, then to prove $B(x, y, p)$ it suffices to assume y is x and p is refl_x . I.e., there is a function

$$\text{path-ind} : \left(\prod_{x:A} B(x, x, \text{refl}_x) \right) \rightarrow \left(\prod_{x,y:A} \prod_{p:x=_A y} B(x, y, p) \right).$$

Arrow induction. If $B(x, y, f)$ is a covariant family dependent on $x, y : A$ and $f : \text{Hom}_A(x, y)$ and A is Segal, then to prove $B(x, y, f)$ it suffices to assume y is x and f is id_x . I.e., there is a function

$$\text{id-ind} : \left(\prod_{x:A} B(x, x, \text{id}_x) \right) \rightarrow \left(\prod_{x,y:A} \prod_{f:\text{Hom}_A(x,y)} B(x, y, f) \right).$$



More theorems about ∞ -categories can be proven using analytic methods in a particular model, but there are other advantages to the synthetic approach:

- **efficiency**: a large part of the theory can be developed simultaneously in many models by working synthetically with ∞ -categories as objects in an ∞ -cosmos.
- **simplification**: the axioms of an ∞ -cosmos are chosen to simplify proofs by working strictly up to isomorphism insofar as possible.
- **model-independence**: ∞ -cosmology may be used to demonstrate that both analytically- and synthetically-proven results about ∞ -categories transfer across suitable “change-of-model” functors.
- **compatible with new foundations**: synthetic constructions can easily be adapted to simplicial HoTT, which yields further streamlining.

References



For more on the synthetic theories of ∞ -categories, see:

Emily Riehl and Dominic Verity

- draft book in progress:

Elements of ∞ -Category Theory

www.math.jhu.edu/~eriehl/elements.pdf

- mini-course lecture notes:

∞ -Category Theory from Scratch

[arXiv:1608.05314](https://arxiv.org/abs/1608.05314)

Emily Riehl and Michael Shulman

- A type theory for synthetic ∞ -categories, Higher Structures 1(1):116–193, 2017; [arXiv:1705.07442](https://arxiv.org/abs/1705.07442)

Thank you!