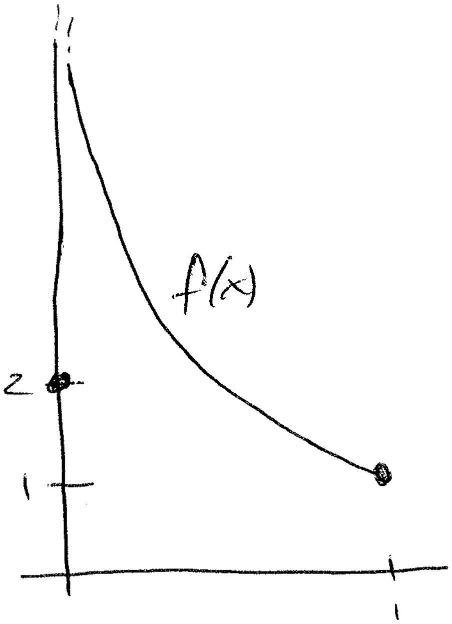
Class 22: 3/30/14 Section 5.1



Extreme Value Thm Notes

(I) $f(x)$ must be continuous.

ex. Let $f(x) = \begin{cases} \frac{1}{x} & x \in (0,1] \\ 2 & x = 0 \end{cases}$

Here f is defined on $[0,1]$ but not continuous at $x=0$. So what is the global max?

(II) Interval must be closed and bounded (finite in length).

ex. Let $f(x) = x^2$ on $[1,2)$. Global max?

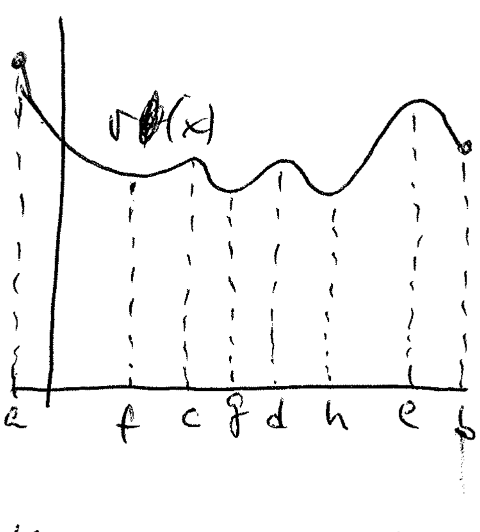
ex. $g(x) = x^2$ on $[1,\infty)$. Global max?

Def $f(x)$ on a domain D has a *local maximum* at $x=c$ if there is a $\delta>0$ so that $f(x)\le f(c)$ for all $x\in(c-\delta, c+\delta)$ in D.

Notes ① Similar def for a minimum.

② The δ does not matter. $f(c)$ just needs to be the extreme value of $f(x)$ "near" $x=c$ on both sides.

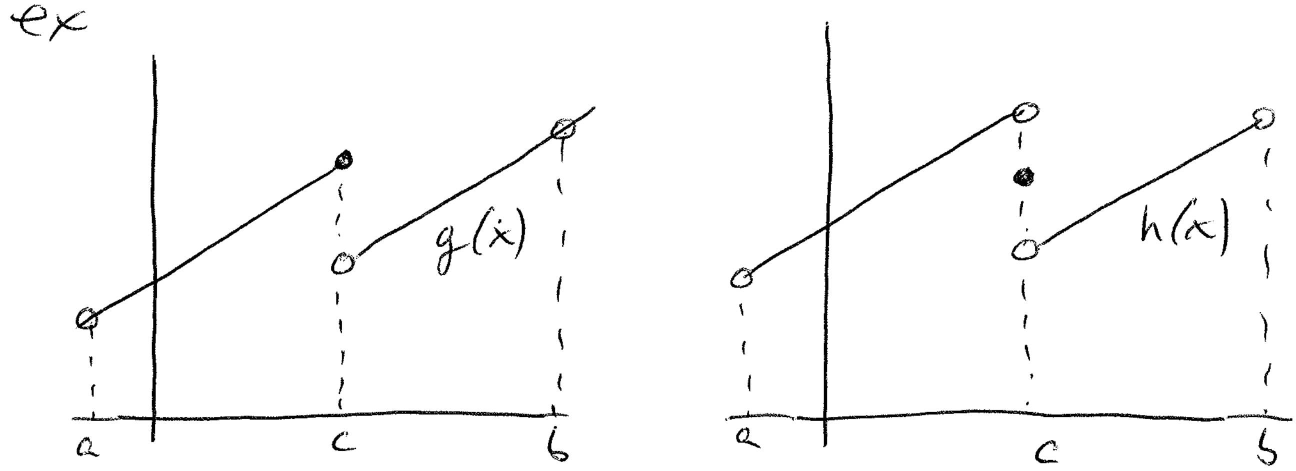
ex.



Here $r(x)$ has a global max at $x=a$ on $[a,b]$, and local max's at $x=c$, $x=d$, and $x=e$.

$r(x)$ has a global min at the smaller value of $f(x)$ at either $x=g$ or $x=h$ and local mins at $x=f$, $x=g$, $x=h$, and $x=b$.

ex



- $g(x)$ has a local max at $x=c$ but no local min. (why not?) on $[a,b]$.
- $h(x)$ has neither a local max or a local min anywhere on $[a,b]$

Note: In applications, local extrema are very <u>important</u>.

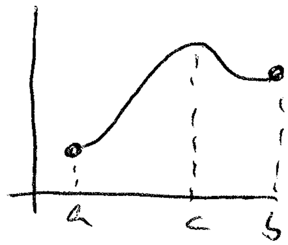
Q: How does one locate extrema? Where would they occur?

A: Think through the previous examples. By the EVT, $f(x)$ on a closed $[a,b]$ will have extrema at 2 possible places:

Extrema of $f(x)$ on $[a,b]$ occur at

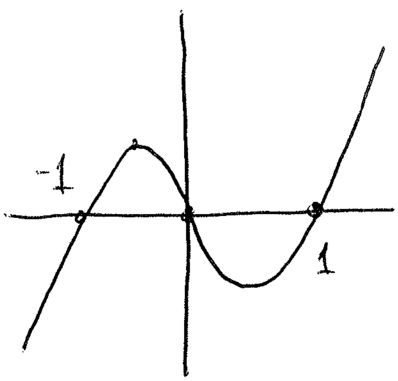
(I) An endpt, or

(II) An interior pt.



Here max at $x=c$, min at $x=a$.

ex. $f(x) = x(x-1)(x+1) = x(x^2-1) = x^3 - x$

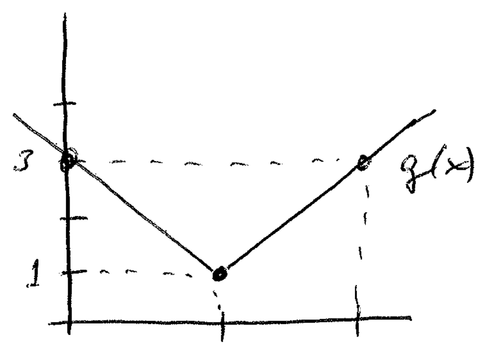


Q: How can we locate the only local max of $f(x)$?

A: $f(x)$ is diff. and max occurs where

$$f'(x) = 0 = \frac{d}{dx}[x^3 - x] = 3x^2 - 1.$$

This is solved by $x = \pm \frac{1}{\sqrt{3}}$



ex. $g(x) = 2|x-1| + 1$

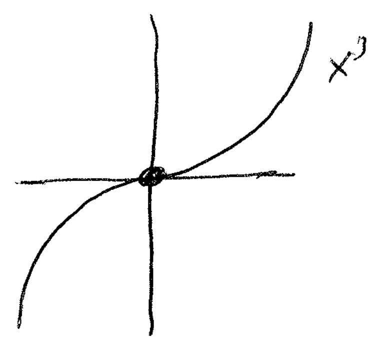
Here the derivative of $g(x)$ where it exists is never 0. However, there is a local min at $x=1$. What are the features of the graph that may help you locate the extrema?

<u>Fermat's Thm</u> (Not his last!).

If f has a local extremum at an interior pt c of an interval, and $f'(c)$ exists, then $f'(c) = 0$.

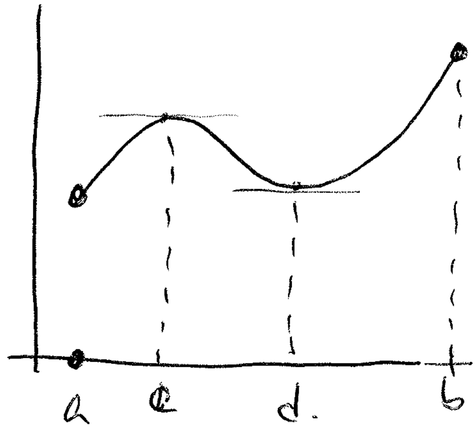
<u>Notes</u> ① The converse is not true; Just because there is a pt c, where $f'(c)$ exists and $f'(c) = 0$, it does not follow that there is an extremum at $x = c$.

ex. $f(x) = x^3$. $f'(0) = 3x^2\big|_{x=0} = 0$ and yet $x = 0$ is not an extremum.



② If $f(x)$ is diff on D, then a global extremum, if it exists, will occur either at an endpt of D or at an interior pt c where $f'(c) = 0$.

ex



Here, $h(x)$ is diff on (a,b) and cont on $[a,b]$. Hence all local extrema will be at four possible places: $x=a$, $x=b$, $x=c$, or $x=d$.

These are the 2 endpts and the 2 interior pts where $h'(x)=0$.

Q: What if there are a few places where $f(x)$ is not diff. Can they be extrema?

<u>Def</u>. A pt c in the domain of $f(x)$ where either $f'(c)=0$ or $f'(c)$ does not exist is called a critical pt of f.

<u>Fact</u> For $f(x)$ continuous on a closed, bounded interval, the global max <u>will</u> occur either at (1) an end pt, or (2) at a critical pt.