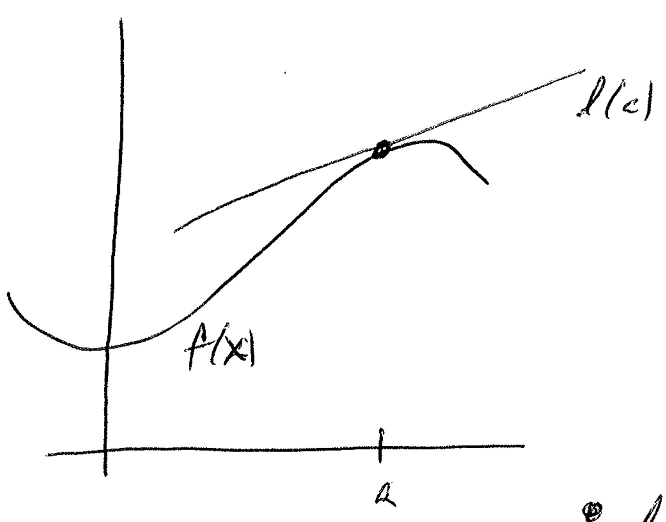
Class 21: 3/28/14 Section 5.1

Q: Suppose you needed to know $\sqrt{110}$ but did not have a calculator. What is your most accurate guess? How can you be accurate?

The tangent line to a pt $(a, f(a))$ on the graph of a function $f(x)$ is called the best linear approximation to the curve at a.



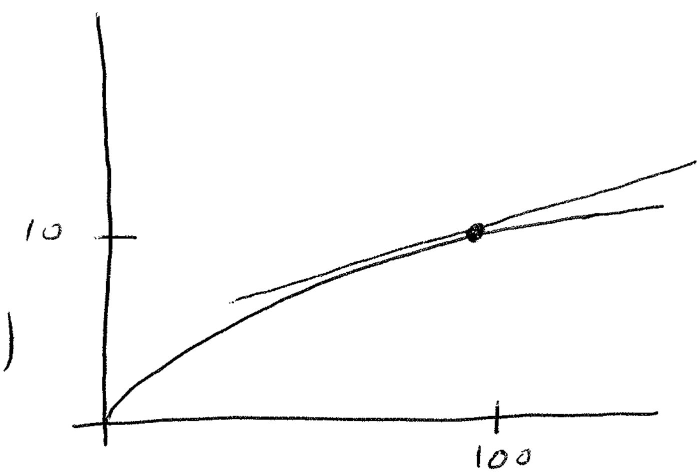
- The equation of the line $l(x) = f(a) + f'(a)(x-a)$
- $l(x)$ has the same value at $x=a$: $l(a) = f(a)$
- $l(x)$ has the same derivative at $x=a$: $l'(a) = f'(a)$.

Hence, for values of x "near" a, $l(x)$ and $f(x)$ are very close.

ex. Let $f(x) = \sqrt{x}$. Here $f(110) = \sqrt{110}$ is the exact value you are looking for. But how to calculate it?

There is a value of $\sqrt{x}$ nearby that is easy to calculate: $f(100) = \sqrt{100} = 10$.

The ~~[illegible]~~ tangent line to $f(x)$ at $x=10$ is



$$l(x) = f(100) + f'(100)(x-100)$$

$$= 10 + \frac{1}{2\sqrt{100}}(x-100)$$

$$= 10 + \frac{1}{20}(x-100) = 10 + \frac{1}{20}x - 5 = \frac{1}{20}x + 5$$

Since the tangent line to $f(x)$ at $x=100$ is close to $f(x)$ at $x=110$, $l(110) \approx f(110)$

$$10.5 = \frac{1}{20}(110)+5 \approx \sqrt{110}$$

Actual value is closer to $f(110) = \sqrt{110} \doteq 10.4881\ldots$

HW Do the same for the known value $x=121$.

Def For $y = f(x)$ differentiable at $x = a$, the line $\ell(x) = f(a) + f'(a)(x-a)$ is called the tangent line approximation or the local linearization of $f(x)$ at $x = a$.

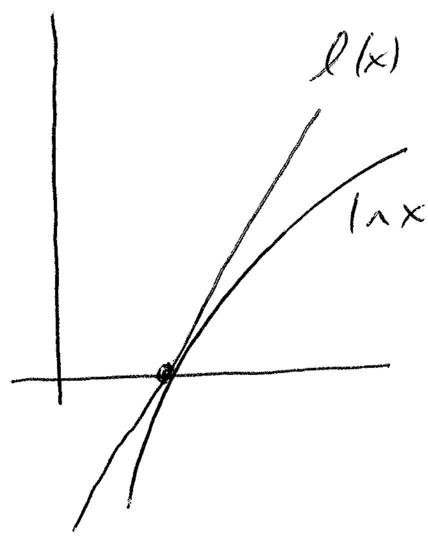
ex. Estimate $\ln(1.1)$ using the tangent line approximation of $f(x) = \ln x$ at $x = 1$.

Solution: Here, $f(1) = 0$ and

$$f'(1) = \left.\frac{1}{x}\right|_{x=1} = 1.$$

Hence

$$\begin{aligned} \ell(x) &= f(1) + f'(1)(x-1) \\ &= 0 + 1(x-1) \\ &= x-1 \end{aligned}$$



And $\cancel{\ell(x)} = \ell(1.1) = 1.1 - 1 = .1 \approx f(1.1) = \ln 1.1$

The actual value is closer to $.09531$.

Note: There are ways to measure the accuracy of the local linearization. We will not do this here.

<u>Def</u> Let f be defined on a domain D and let $c \in D$. Then f has a <u>global</u> <u>maximum</u> at $x=c$, if $f(x) \leq f(c)$ for all $x \in D$. $f(x)$ has a <u>global</u> <u>minimum</u> at $x=c$ if $f(x) \geq f(c)$ for all $x \in D$.

Note: ① A maximum or a minimum is also called an <u>extremum</u>.

② The domain D will be very important here.

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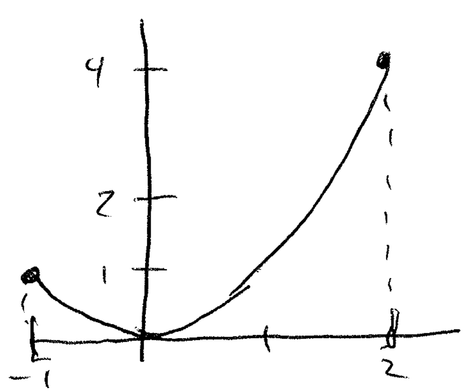
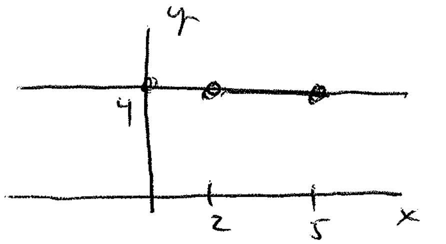
ex. Let $f(x) = x^2$ on $D = [-1,2]$. Then f has a global maximum at $x=2$ and a global minimum at $x=0$.



Note: f has $x=1$ as a global min but <u>No</u> global max on $(-1,2)$.

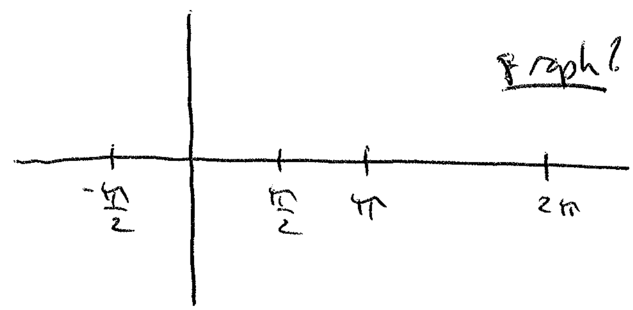
ex. $g(x) = 4$ on $D = [2, 5]$.



Where is (are) the global extrema?

ex. $h(x) = \sin x$ on $[-\frac{\pi}{2}, 2\pi]$. Where are the global extrema?

graph?



How about on $(-\frac{\pi}{2}, 2\pi)$?

Q: Are there ways of knowing when a function will have extrema? A: Sometimes

Extreme Value Thm

If $f(x)$ is continuous on a closed finite interval $[a, b]$, where $-\infty < a < b < \infty$, then f has a global minimum and a global max.