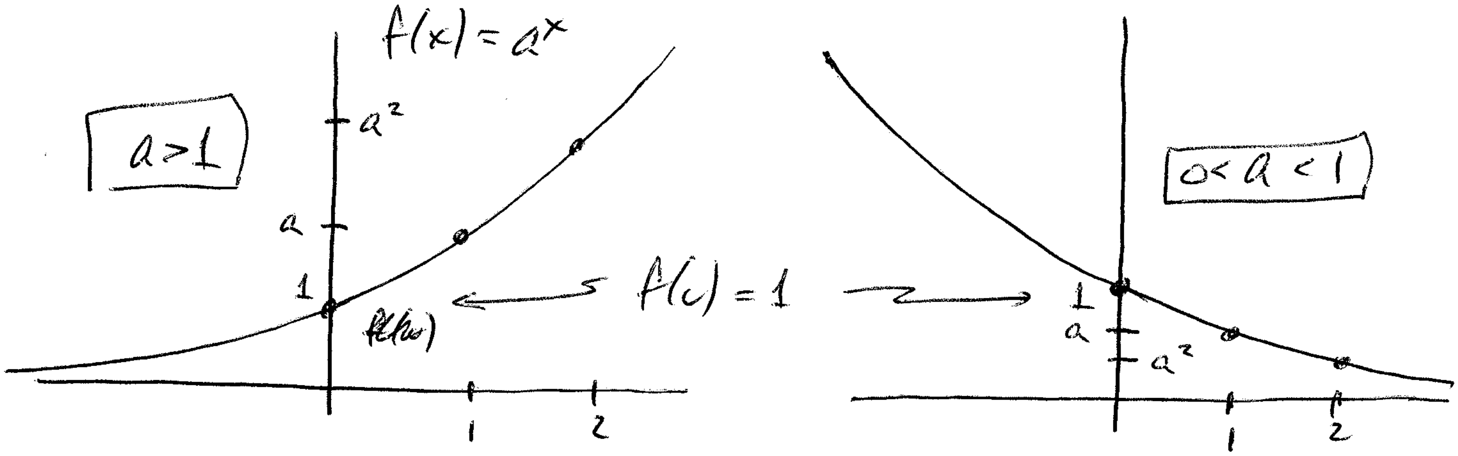
Class 3: 1/31/14 Section 2.1 I

Continuous vs. discrete time functions

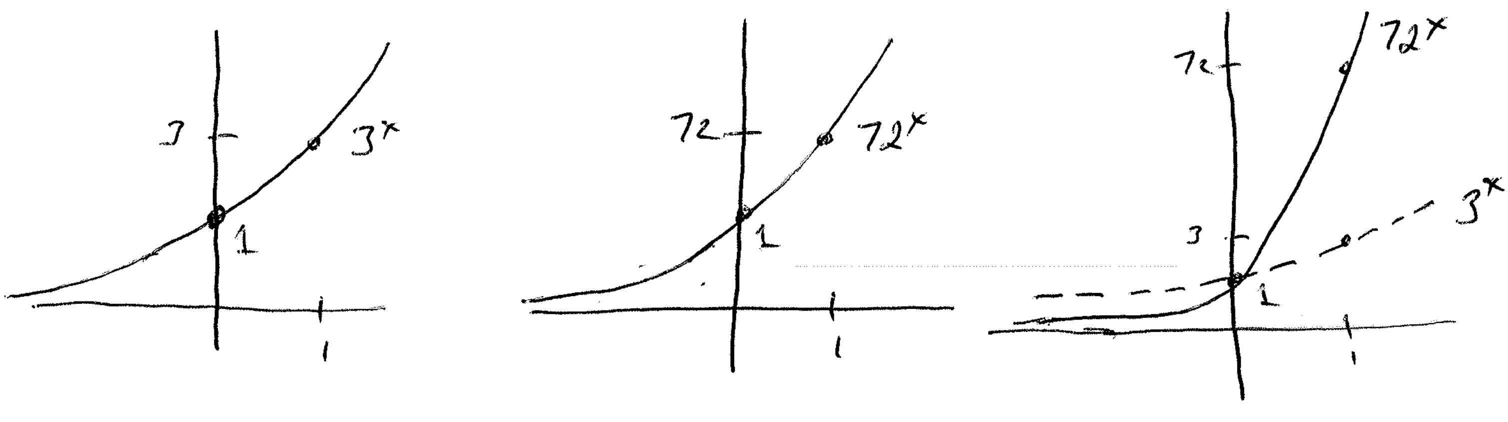
Go back to exponential functions



exponential growth exponential decay

In both cases, $f: \mathbb{R} \to \mathbb{R}$ where the domain is all of $\mathbb{R}$ (any value of $x \in \mathbb{R}$ is valid), and the range is only $\mathbb{R}_+ = \{y \in \mathbb{R} \mid y > 0\}$. why? (is it possible that there is an $x \in \mathbb{R}$, where $f(x) = a^x \overset{?}{=} 0$, or $f(x) = a^x \overset{?}{<} 0$).

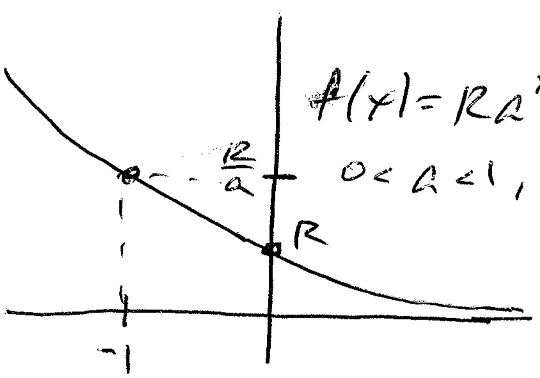
Note how graph changes for say $a = 3$ and $a = 72$:

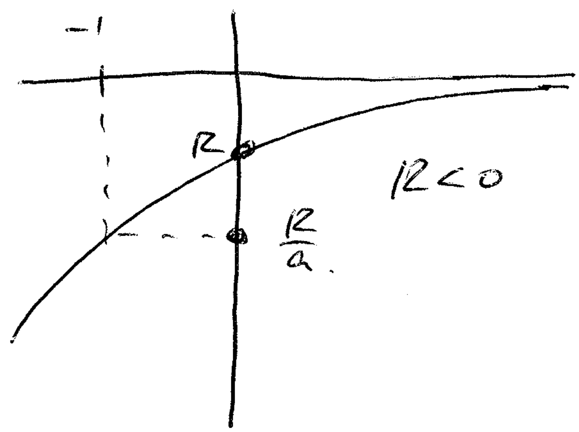


The graphs are the same in shape and properties, although when plotted on the same graph are different relative to each other.

Take care to understand the visual nature of functions rather than simply where the points are located.

Q: How does multiplication by a nonzero constant R affect a graph of an exponential func?





In many applications, (radio-carbon dating, population dynamics, etc) model functions are exponential, with

- a determining growth ($a>1$) or decay ($0<a<1$)
- x is usually time
- k is some initial amount (why?)

and x is called a <u>continuous variable</u> (there is an output for every real x input, at least on some interval of $\mathbb{R}$).

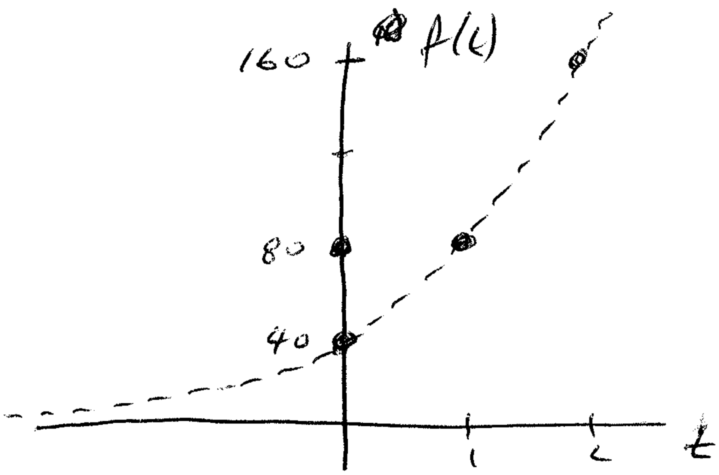
ex. Section 1.2, #58. $N(t) = 40 \cdot 2^t$, $t \geq 0$.

Here $t \in [0, \infty)$.

Sometimes for population dynamics (especially when populations are small), t (or x) may only be defined for discrete values of $\mathbb{R}$:

ex. Altered section 1.2 #58

$f: \mathbb{N} \to \mathbb{R}$, $f(t) = 40 \cdot 2^t$, $t \in \mathbb{N}$.



Here, t measures time in some units and it is not obvious what the intervals are until specified.

ex. $f(t) = 40 \cdot 2^t$, $t \in \mathbb{N}$ measures a population of bacteria where t is a 20 minute interval (ex, pg 62, w/ $N_0 = 40$).

- at $t=0$ $N(0) = 40$
- at $t=1$ $N(1) = 40 \cdot 2 = 80$

  There are 80 bacteria after 20 minutes.
- at $t=5$ $N(5) = 40 \cdot 2^5 = 1280$

  There are 1280 bacteria at the 1:40 hour mark.

ex. $g(t) = 40{,}000 \cdot 3^t$, $t \in \mathbb{N}$ ~~does~~ measured in hours.

~~ex.~~ Here population starts at 40,000 and triples every hour.

ex. $h(x) = 32000\left(\frac{1}{2}\right)^x$ x measured in 6 month intervals.

Q: When does population fall below 1000?