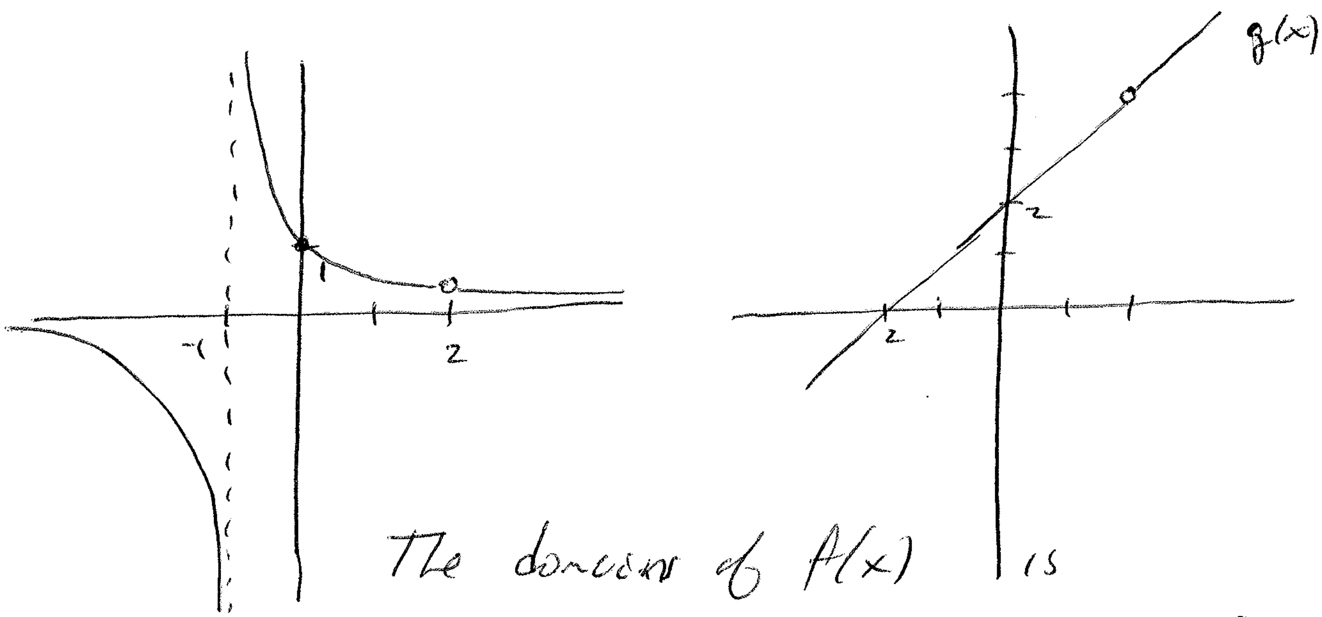
# Class 8: 2/12/14 Section 3.2

To start, here are 2 examples:

ex. $f(x) = \dfrac{x-2}{x^2-x-2}$, and $g(x) = \dfrac{x^2-4}{x-2}$.

Sketch their graphs and find $\lim_{x\to 2} f(x)$, $\lim_{x\to -1} f(x)$, and $\lim_{x\to 2} g(x)$, if they exist.




The domain of $f(x)$ is $\{x \in \mathbb{R} \mid x \neq 2, -1\}$ and $g(x)$ is $\{x \in \mathbb{R} \mid x \neq 2\}$.
because of the divide by zero problem (rational functions where denominator is 0).

For $f(x)$, notice that as long as $x \neq -1, 2$,
$f(x) = \dfrac{x-2}{x^2-x-2} = \dfrac{x-2}{(x-2)(x+1)} = \dfrac{1}{x+1}$. Hence the graph is easy to see.

Caution, however, that the pt $x=2$ is still
<u>NOT</u> in the domain of $f(x)$.

For $g(x)$, we see that as long as $x \neq 2$,

$$g(x) = \frac{x^2-4}{x-2} = \frac{(x-2)(x+2)}{x-2} = x+2 \text{ on all } x \neq 2.$$

Now discuss whether the limits exist.

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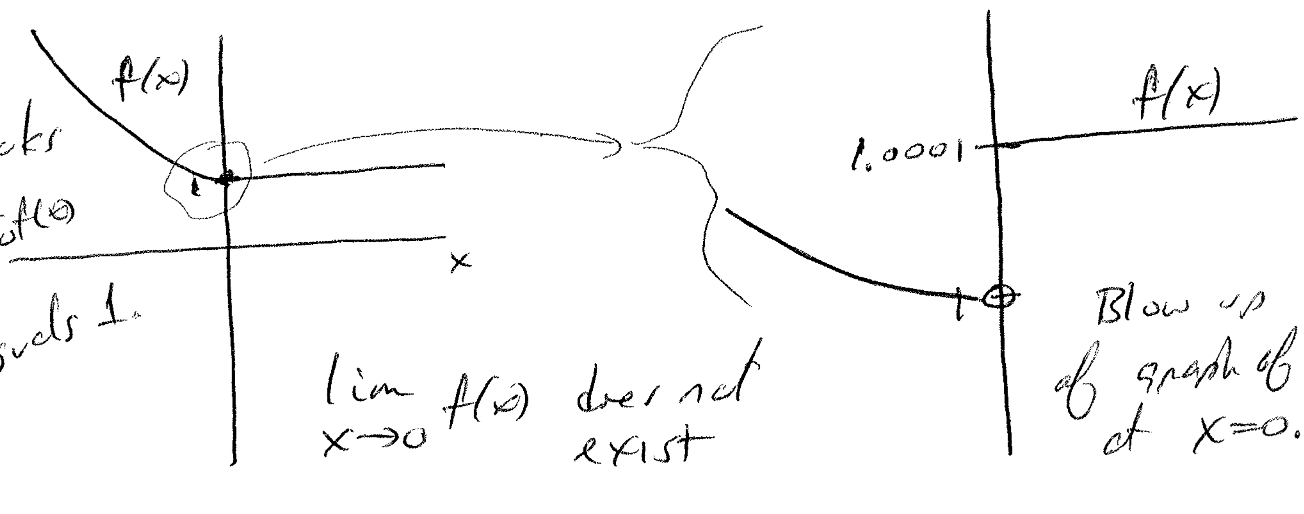
Section 3.1 uses tables and graphs to
help determine limits
Visual aids are nice (calculators and computers)
but be very careful. They can be tricky.

ex: $\lim_{x \to 0} \sin \frac{4\pi}{x}$ (ex 8, pg 97)

ex: $\lim_{x \to 0} f(x)$ where $f(x) = \begin{cases} e^{-x} & x<0 \\ 1.0001 & x \geq 0 \end{cases}$

Here, it looks like $\lim_{x\to 0} f(x)$ exists and equals 1.




$\lim_{x \to 0} f(x)$ does not exist

Blow up of graph of $f(x)$ at $x=0$.

Even though limits may be tricky or difficult to discern in certain situations, most functions are well-behaved and have nice properties that are easy to identify.

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## One sided limits

<u>Def</u> A function $f(x)$ has a limit $L$ "from the right" at $x=c$, denoted $\lim_{x\to c^+} f(x) = L$ if limit exists on the right side of $c$.

Note: Similar def "from the left," $\lim_{x\to c^-} f(x) = L$.

<u>Thm</u> Given $f(x)$ defined near a pt $c\in\mathbb{R}$ but possibly not at $x=c$, if $\lim_{x\to c^-} f(x) = a$ and $\lim_{x\to c^+} f(x) = b$, then

$$\lim_{x\to c} f(x) = L \quad \text{iff} \quad a=b=L.$$

Final notes

① The limit Laws on pg 98 will be your primary means of finding limits of complicated functions once the easy ones are known

② For the elementary functions of Chapter 1:

power fncs, polynomials, rational functions, exponential fncs, logarithmic, trigonometric,

for each pt $c$ in the domain of function,

$$\lim_{x \to c} f(x) = f(c).$$

This is a special (and nice) property.

**Def** A function $f(x)$ is said to be continuous at $x=c$ if $\lim_{x \to c} f(x) = f(c)$.

# Notes on continuity

① This is exactly the notion that ~~the~~ the graph of $f(x)$ can be drawn "through" an input value $c$ "without lifting the pencil from the paper".

② All elementary functions of Chapter 1 are continuous on their domains.

Special Note: It may sound strange, but when restricted to its domain, a rational function will satisfy $\lim_{x \to c} f(x) = f(c)$ for every $c$ in its domain (the pts of $f(x) = \frac{p(x)}{q(x)}$ where $q(x) = 0$ are NOT in the domain). Hence rational functions are continuous for every pt in their domain.

ex. In the examples $f(x) = \frac{x-2}{x^2-x-2}$, $g(x) = \frac{x^2-4}{x-2}$

f and g are continuous functions on their domains.

③ A function $f(x)$ which is continuous at every pt in its domain is said to be a continuous function on that domain.

ex. $g(x) = \frac{x^2-4}{x-2}$ is continuous on $\{x \in \mathbb{R} \mid x \neq 2\}$.

$h(x) = \begin{cases} g(x) & x \neq 2 \\ 1 & x = 2 \end{cases}$ is not continuous on $\mathbb{R}$.
<u>why?</u>

$i(x) = \begin{cases} g(x) & x \neq 2 \\ 4 & x = 2 \end{cases}$ is continuous on $\mathbb{R}$
<u>why?</u>

④ Continuity needs three criteria:

(B) For $\lim_{x \to c} f(x) = f(c)$, we need

(I) c must be in domain of f

(II) $\lim_{x \to c} f(x)$ must exist

(III) limit must equal $f(c)$.

⑤ If $f(x)$ is not continuous at $x = c$, then we say f is <u>discontinuous</u> at $x = c$.

⑥ What happens when a domain has an edge pt?

<u>Def</u> A function is continuous on the domain $[a,b]$ if

① $\lim_{x\to c} f(x) = f(c)$ for all $c \in (a,b)$

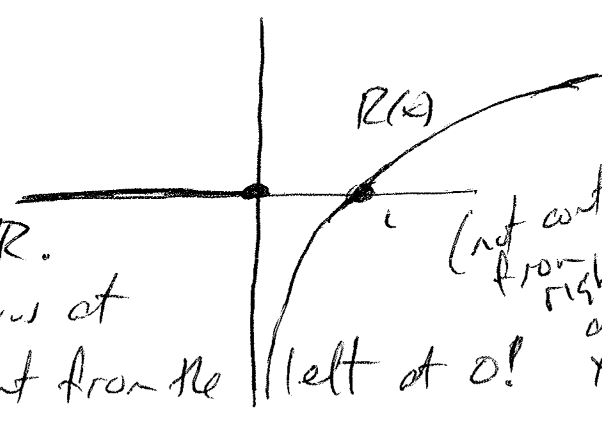
② $\lim_{x\to a^+} f(x) = f(a)$ called continuous from the right at $a$.

③ $\lim_{x\to b^-} f(x) = f(b)$ called continuous from the left at $b$.

ex. Given $\ell(x) = \ln x$, we have

- $\ell(x)$ is continuous on $[1,\infty)$
- $\ell(x)$ is continuous on $(0,\infty)$
- $\ell(x)$ is not continuous on $[b,\infty)$, for any $b \le 0$.

ex. $R(x) = \begin{cases} \ln x & x > 0 \\ 0 & x \le 0 \end{cases}$




(not cont from right at $x=0$.)

Domain of $R(x)$ is all $\mathbb{R}$.
But $R(x)$ is discontinuous at $x=0$, though cont from the left at 0!

(7) The product, sum, difference, and ratio of 2 continuous functions is continuous (wherever the domain makes sense, that is).

ex. $p(x) = 1+x$ and $q(x) = x$ are continuous functions on all $\mathbb{R}$.

- So is $p(x)+q(x) = 1+2x$
- So is $p(x)\,q(x) = x+x^2$
- But $\dfrac{p(x)}{q(x)} = \dfrac{x+1}{x} = 1+\dfrac{1}{x}$ is only continuous on $\{x \in \mathbb{R} \mid x \neq 0\}$.