Class 23: 4/2/14 Section 5.2

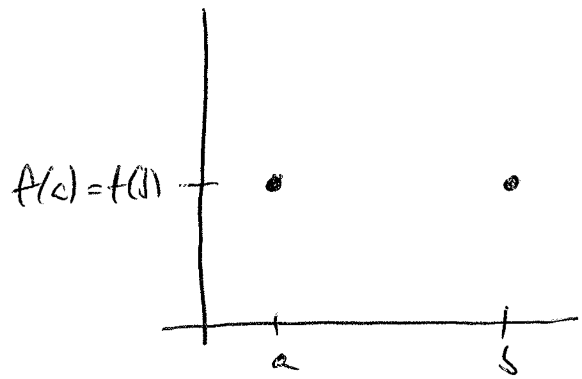
I

2 big idea's

① Rolle's Thm

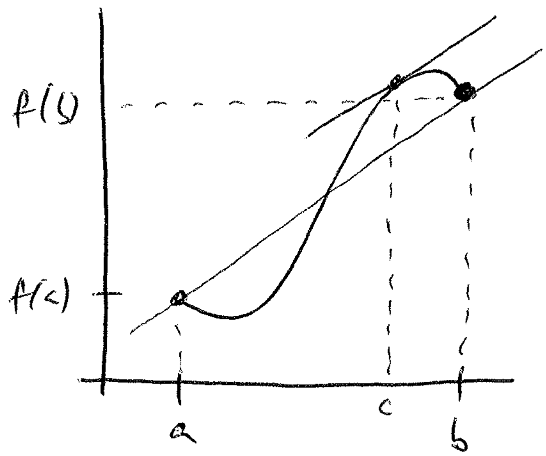
Suppose $f(x)$ is continuous on $[a,b]$ and diff. on (a,b), where $f(a)=f(b)$.

Then there exists a pt $c \in (a,b)$ where $f'(c)=0$



pf. EVT says there must be a max. If it is in (a,b) then by Fermat, f' is 0 here. If it is at an end pt, then there must be a min. in the interior (why?). By Fermat again, derivative must be 0 at this pt ∎

Mean Value Thm [Inclined Rolle's Thm].



If $f(x)$ is continuous on $[a,b]$ and diff on (a,b), then there is a pt $c \in (a,b)$ where $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

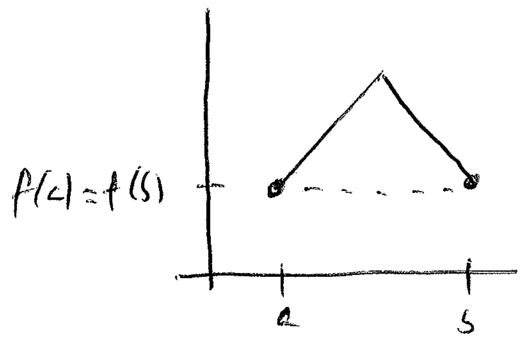
pf. is straight forward and in the book.

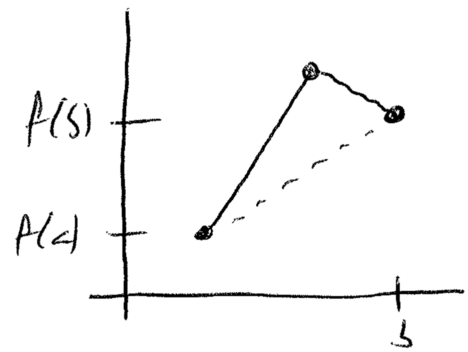
Notes ① In both of these thms, the average rate of change of $f(x)$ on $[a,b]$ is $\dfrac{f(b) - f(a)}{b - a}$. Thus both thms say that somewhere in (a,b), the instantaneous rate of change of $f(x)$ must equal the average rate of change on $[a,b]$.

Instantaneous rate of change at $c \in (a,b)$ — Average rate of change over all $[a,b]$.

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Notes ② MVT and Rolle's Thm do not work
cont'd. if f is not differentiable on (a,b)





avg rate of change on $[a,b]$ is 0, but no place c, where $f'(c)=0$

average rate of change is $\frac{f(b)-f(a)}{b-a}$ but no place c where $f'(c)$ equals this quantity.

③ If you travel the entire New Jersey Turnpike (122 miles) in 90 minutes, it must be the case that at some point you were going $\frac{122 \text{ miles}}{1.5 \text{ hours}} = 81+ \text{ miles/hour}$.

Electronic Ticket, maybe (entry tickets, and EZPass are time-stamped).

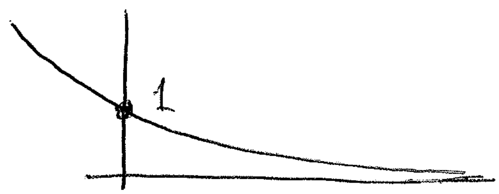
Do keep in mind that Rolle's Thm is just the MVT with $f(b)=f(a)$.

<u>Application</u> The von Bertalanffy Eqn. is a model for fish populations.

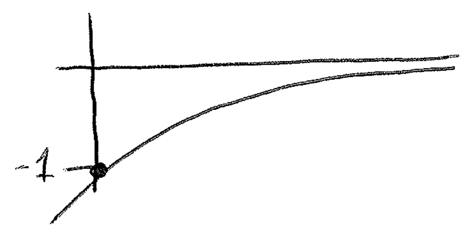
Graph $L(x) = L_\infty - (L_\infty - L_0)e^{-kx}$ when

$k>0$, $L_\infty > L_0 > 0$.

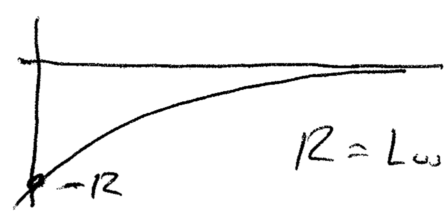
Here (a) e^{-kx}, $k>0$



(b) $-e^{-kx}$, $k>0$

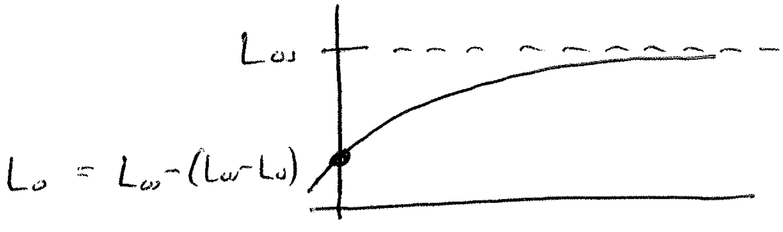


(c) $-Re^{-kx}$, $R, k>0$



$R = L_\infty - L_0 > 0$

(d) $L_\infty - Re^{-kx}$, $L_\infty, R, k > 0$



Def A function f on an interval I is called (strictly) increasing on I if

$$f(x_1) < f(x_2) \text{ whenever } x_1 < x_2 \text{ in } I$$

Similar for (strictly) decreasing

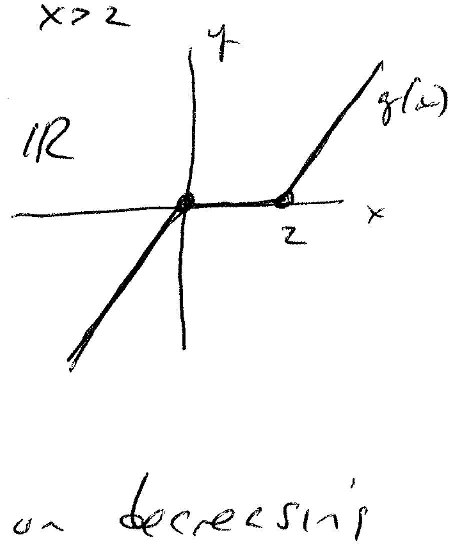
Notes ① Strictly means "$<$" as opposed to "$\leq$"

ex $f(x) = 2x$ is strictly increasing on $[0, 10]$

while $g(x) = \begin{cases} 2x & x < 0 \\ 0 & 0 < x < 2 \\ 2x-4 & x > 2 \end{cases}$ is not

strictly increasing on $\mathbb{R}$

(we would call $g(x)$ "non decreasing").



② A strictly increasing or decreasing function is also called monotonic.

<u>Criteria for Monotonicity</u> Let f be cont on $[a,b]$ and diff on (a,b)

ⓐ If $f'(x) > 0$ for all $x \in (a,b)$, then f is increasing on $[a,b]$.

ⓑ If $f'(x) < 0$ for all $x \in (a,b)$ then f is decreasing on $[a,b]$.

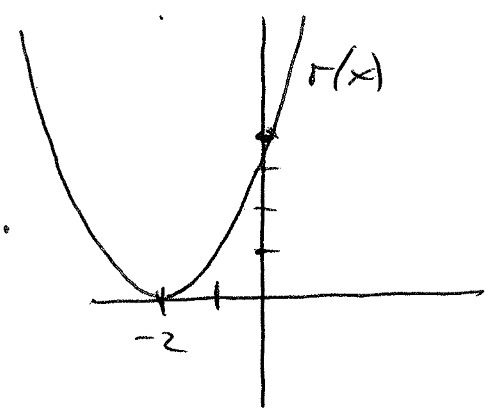
ex. $h(x) = e^{2x}$. Here $h'(x) = 2e^{2x} > 0$ everywhere.

Hence $h(x)$ is increasing on $\mathbb{R}$.

ex. $r(x) = x^2 + 4x + 4$. Here $r'(x) = 2x + 4$.

$r'(x) > 0$ on $[-2, \infty)$. Hence $r(x)$ is increasing there.

And $r'(x) < 0$ on $(-\infty, -2]$. Hence $r(x)$ is decreasing there. Of course, this is obvious from the graph. But if you did not have the graph?



ex. $L(x) = L_\infty - (L_\infty - L_0)e^{-kx}$, $L_\infty, L_0, k > 0$
$L_\infty > L_0$.

Here $L'(x) = \underbrace{k}_{>0}\underbrace{(L_\infty - L_0)}_{>0}\underbrace{e^{-kx}}_{>0} > 0$ on $[0, \infty)$.

$L(x)$ is always increasing!