HW7 solutions

4.4 / 16, 32, 34, $\boxed{38}$, 44, 50, $\boxed{56}$, 62, 68, $\boxed{70}$, 71, 80, 84, 86

4.5 / $\boxed{14}$, 18, 28, 50, 58, 60, $\boxed{61}$, 72, 73

$\boxed{4.4.38}$ Q: Assume that $f(x)$ and $g(x)$ are differentiable. Find $\frac{d}{dx} f\left[\frac{1}{g(x)}\right]$

A: Chain Rule here, with our outer function equal to $f(x)$ and our inner function equal to $\frac{1}{g(x)}$ gives:

$$\frac{d}{dx} f\left[\frac{1}{g(x)}\right] = f'\left[\frac{1}{g(x)}\right] \cdot \underbrace{\frac{-1}{(g(x))^2} \cdot g'(x)}$$

where our derivative of the inner function $\frac{1}{g(x)}$ was taken using chain rule again

$$\frac{1}{g(x)} = f_1(g_1(x)) \qquad \begin{aligned} f_1(x) &= \frac{1}{x} \\ g_1(x) &= g(x) \end{aligned}$$

$\boxed{4.4.56}$ Q: Find the lines that are tangential and normal to

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

at

$$\left(1, \tfrac{3}{2}\sqrt{3}\right)$$

A: We have a point, so the only information left to discern is the slope, $\frac{dy}{dx}$.

Implicit differentiation gives:

$$\frac{d}{dx}\left(\frac{x^2}{4} + \frac{y^2}{9}\right) = \frac{d}{dx} 1$$

$$\frac{2x}{4} + \frac{2y}{9}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2x}{4} \cdot \frac{9}{2y} = \frac{-9x}{4y}$$

Plug in our point

$$\frac{dy}{dx} = \frac{-9}{6\sqrt{3}} = -\frac{\sqrt{3}}{2}$$

Thus, the tangent line is

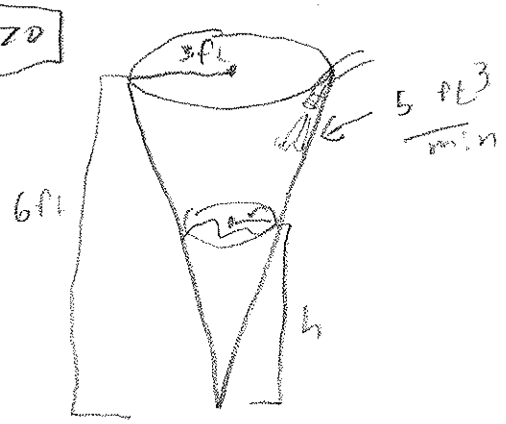
$$y - \tfrac{3}{2}\sqrt{3} = -\frac{\sqrt{3}}{2}(x-1)$$

And the normal line (slope = negative reciprocal) is:

$$y - \tfrac{3}{2}\sqrt{3} = \frac{2\sqrt{3}}{3}(x-1$$

4.4.70

Q:



water is pumped into the cone at $5 \frac{ft^3}{min}$.

The tank is of 3ft radius and 6ft height

what is the rate that the water is rising when it is 2ft deep

A: Mathematical interpretation:

$\frac{dV}{dt} = \frac{5 ft^3}{min}$, At $h = 2ft$, we want to solve for $\frac{dh}{dt}$

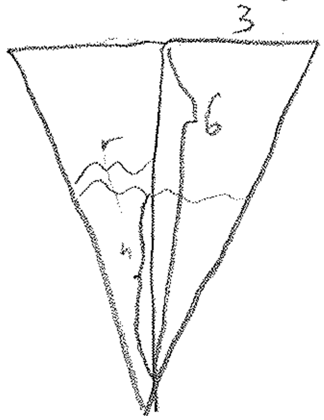
we have $V = \frac{\pi}{3} r^2 h$, so differentiating by $\frac{d}{dt}$ helps

$$\frac{dV}{dt} = \frac{\pi}{3}\left(2rh\frac{dr}{dt} + r^2\frac{dh}{dt}\right)$$ (product rule)

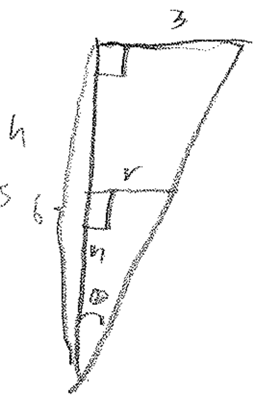
Plugging in what we know gives

$$5 = \frac{\pi}{3}\left(4r\frac{dr}{dt} + r^2\frac{dh}{dt}\right)$$

To solve we need r, and $\frac{dr}{dt}$ when $h=2$. For that, we need another two equations. we have that at any given t, the cross section of the cone is:



which gives us



a pair of similar triangles!

From this, we have

$$\frac{r}{3} = \frac{h}{6} \longrightarrow 6r = 3h \longrightarrow 2r = h$$

$2r = h$ gives us that at $h = 2$,

$$r = 1$$

We still need $\frac{dr}{dt}$, but differentiating gives

$$2\frac{dr}{dt} = \frac{dh}{dt} \longrightarrow \frac{dr}{dt} = \frac{1}{2}\frac{dh}{dt}$$

Our original equation again:

$$5 = \frac{\pi}{3}\left(4r\frac{dr}{dt} + r^2\frac{dh}{dt}\right)$$

is now

$$5 = \frac{\pi}{3}\left(2\frac{dh}{dt} + \frac{dh}{dt}\right) = \pi\frac{dh}{dt}$$

So $$\left.\frac{dh}{dt}\right|_{h=2} = \frac{5}{\pi}\ \frac{ft}{min}$$

$\boxed{4.5.14}$ Q: Find $\frac{d}{dx}(-3\csc(3-5x))$

A: Chain rule $f(x) = -3\csc(x)$
$g(x) = 3-5x$

$$\frac{d}{dx}(-3\csc(3-5x)) = 3\csc(3-5x)\cot(3-5x)\cdot -5$$

Because $\frac{d}{dx}\csc x = -\csc x \cdot \cot x$

$\boxed{4.5.61}$ Q: Use the identity $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

and the definition of the derivative to show $\frac{d}{dx}\cos x = -\sin x$

A: $$\frac{d}{dx}\cos x = \lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h}$$

$$= \lim_{h\to 0}\frac{\cos x\cos h - \sin x\sin h - \cos x}{h}$$

As $h \to 0$ $\cos h \to 1$, so $\cos x\cos h - \cos x \to 0$ and the other term dominates

$$= \lim_{h\to 0}\frac{-\sin x\sin h}{h} = -\sin x\lim_{h\to 0}\frac{\sin h}{h} = -\sin x(1)$$

$$= -\sin x$$