Selected Homework Solutions: Problem Set I

4.2: 8, 16, 22, 26, 30, 36, 40, 56, 66, 70, 74, 80, 82

4.3: 6, 18, 28, 36, 50, 58, 78, 83, 86

Problem 4.3.22 Find $f'(x)$, for $f(x) = \pi x e^2 - \frac{x^2\pi}{e}$.

Strategy: As $f(x)$ is a function whose sole variable is x, the symbols π and e are simply numbers. Here $f(x)$ is a polynomial in x. We calculate using the Power, Sum, and Constant Mult. Rules.

Solution: As a polynomial in x,

$$f(x) = (\pi e^2)x - \left(\frac{\pi}{e}\right)x^2.$$

$$\text{Here } \frac{d}{dx}[f(x)] = \frac{d}{dx}\left[(\pi e^2)x\right] - \frac{d}{dx}\left[\left(\frac{\pi}{e}\right)x^2\right]$$

$$= \pi e^2 \frac{d}{dx}(x) - \frac{\pi}{e}\frac{d}{dx}[x^2]$$

$$= \pi e^2 \cdot 1 - \frac{\pi}{e}(2x)$$

$$\boxed{f'(x) = \pi e^2 - \frac{2\pi}{e}x}$$

The derivative of $f(x) = \pi x e^2 - \frac{x^2\pi}{e}$ is $f'(x) = \pi e^2 - \frac{2\pi}{e}x$.

Problem 4.2.44 Find the tangent line to $y = f(x) = -2x^3 - 3x + 1$ at the pt $x = 1$.

Strategy: We calculate $f'(1)$ and then use the pt $(1, f(1))$ to create the equation of the tangent line.

Solution: We use a combination of Power, Constant Multiple and Sum Rules to calculate $f'(x)$:

$$f'(x) = \frac{d}{dx}[-2x^3] + \frac{d}{dx}[-3x] + \frac{d}{dx}[1]$$
$$= -6x^2 - 3$$

So that $f'(1) = -6(1)^2 - 3 = -9$

And since $f(1) = -2(1)^3 - 3(1) + 1 = -4$, we set

$$y - f(1) = f'(1)(x-1)$$
$$y - (-4) = -9(x-1), \text{ or}$$
$$y = -9x + 9 - 4 \Rightarrow \boxed{y = -9x + 5}$$

Problem 4.2.74 – Find a pt on the curve $y = 1-3x^3$ whose tangent line is parallel to the line $y=-x$. Find all points like this.

Strategy: Find all pts where derivative is -1.

Solution: For $y = 1-3x^3$, $\frac{dy}{dx} = -9x^2$. Here if $y' = -1$, then x solves $-1 = -9x^2$, or $x = \pm\frac{1}{3}$. When $x = \frac{1}{3}$, $y = 1-3\left(\frac{1}{3}\right)^3 = \frac{8}{9}$.

and when $x = -\frac{1}{3}$, $y = 1-3x^3 = 1-3\left(-\frac{1}{3}\right)^3 = \frac{10}{9}$.

Hence the 2 pts on the curve $y = 1-3x^3$ where the tangent line is parallel to $y=-x$ are $\left(\frac{1}{3}, \frac{8}{9}\right)$ and $\left(-\frac{1}{3}, \frac{10}{9}\right)$.

4.3.44 Differentiate $y = [-2f(x) - 3g(x)]\,g(x) + \frac{2g(x)}{3}$.

where f, g are differentiable.

<u>Strategy</u>: We will need the Product Rule among others.

<u>Solution</u>: Calculate directly:

$$y' = \frac{d}{dx}\Big[[-2f(x) - 3g(x)]\,g(x) + \tfrac{2}{3}g(x)\Big]$$

$$= \frac{d}{dx}[-2f(x) - 3g(x)] \cdot g(x) + [-2f(x) - 3g(x)]\frac{d}{dx}[g(x)]$$

$$+ \frac{d}{dx}\left[\tfrac{2}{3}g(x)\right].$$

$$y' = [-2f'(x) - 3g'(x)]\,g(x) + [-2f(x) - 3g(x)]\,g'(x)$$

$$+ \tfrac{2}{3}g'(x).$$ This is the derivative.

There is no need to simplify.

4.3.83 Given that $f(2)=-4$, $g(2)=3$, $f'(2)=1$, and $g'(2)=-2$, find $\left(\frac{f}{2g}\right)'(2)$.

Strategy: Compute the derivative using the Quotient Rule, and then substitute in all of the values of the pieces at $x=2$.

Solution: Here $\frac{d}{dx}\left[\frac{f(x)}{2g(x)}\right] = \left(\frac{f}{2g}\right)'(x)$

$$= \frac{f'(x)(2g(x)) - f(x)(2g'(x))}{[2g(x)]^2}$$ So that at $x=2$.

$$\frac{d}{dx}\left[\frac{f(x)}{2g(x)}\right]\Bigg|_{x=2} = \left(\frac{f}{2g}\right)'(2) = \frac{f'(2)(2g(2)) - f(2)(2g'(2))}{[2g(2)]^2}$$

$$= \frac{1(2(3)) - (-4)(2(-2))}{[2(3)^2]^2}$$

$$= \frac{6 - 16}{324} = \boxed{\frac{-10}{324} = \left(\frac{f}{2g}\right)'(2)}$$

4.3.58 Differentiate $f(s) = \dfrac{2s^3 - 4s^2 + 5s - 7}{(s^2-3)^2}$

Strategy: Multiply out denominator and use Quotient Rule, using Power, Sum, and Constant Multiple Rules on each of numerator and denominator.

Solution: For $f(s)$ as above

$$\frac{df}{ds} = f'(s) = \frac{\frac{d}{ds}[2s^3-4s^2+5s-7](s^4-6s^2+9) - (2s^3-4s^2+5s-7)\frac{d}{ds}[s^4-6s^2+9]}{(s^4-6s^2+9)^2}$$

$$f'(s) = \frac{(6s^2-8s+5)(s^4-6s^2+9) - (2s^3-4s^2+5s-7)(4s^3-12s^2)}{(s^4-6s^2+9)^2}$$

There is no need to simplify.

Problem

4.3.78: Assuming $a, k > 0$ are positive constants, diff $f(x) = \frac{ax^2}{k^2+x^2}$ with respect to x.

Solution: Directly $f'(x) = \dfrac{\frac{d}{dx}[ax^2](k^2+x^2) - (ax^2)\frac{d}{dx}[k^2+x^2]}{(k^2+x^2)^2}$

using the Quotient Rule. Thus

$$f'(x) = \frac{(2ax)(k^2+x^2) - (ax^2)(2x)}{(k^2+a^2)^2}$$

There is no need to simplify.

4.3.86 Find an expression for the derivative of $y = [f(x)]^2 - \frac{x}{f(x)}$ at $x=2$, knowing that $f(2) = -1$, $f'(2) = 1$.

Solution: Since $y = f(x)f(x) - \frac{x}{f(x)}$,

$$\frac{dy}{dx} = \frac{d}{dx}[f(x)f(x)] - \frac{d}{dx}\left[\frac{x}{f(x)}\right] = f'(x)f(x) + f(x)f'(x) - \left(\frac{1 \cdot f(x) - xf'(x)}{(f(x))^2}\right)$$

$$y' = 2f'(x)f(x) - \frac{f(x) - xf'(x)}{[f(x)]^2}.$$

4.3.86 (cont'd).

and at $x=2$, we have

$$y'(2) = 2f'(2)f(2) - \frac{f(2) - 2f'(2)}{(f(2))^2}$$

$$= 2(1)(-1) - \frac{(-1) - 2(1)}{(-1)^2} = -2 - \left(\frac{-3}{1}\right) = 5.$$