3.3

6)

$$\lim_{x\to\infty} \frac{1-5x^3}{1+3x^4}$$

From page 111 we have the rule:

$$\lim_{x\to\infty} f(x) = \lim \frac{p(x)}{q(x)} = \begin{cases} 0 & \text{if } \deg(p) < \deg q \\ L \neq 0 & \text{if } \deg(p) = \deg q \\ DNE & \text{if } \deg(p) > \deg q \end{cases}$$

Where L is the ratio of the leading terms.

For our ratio the denomiator ~~has~~ has higher degree than the numerator, so

$$\lim_{x\to\infty} \frac{1-5x^3}{1+3x^4} = 0$$

25) The monoid growth function is

$$r(N) = \frac{aN}{k+N} \qquad N \geq 0$$

Find $\lim_{N\to\infty} r(N)$

We can apply the rule again. the numerator and denominator are both linear, so the limit is the ratio of the leading terms, $\frac{a}{1} = a$

$$\lim_{N\to\infty} \frac{aN}{k+N} = a$$

13)
$$\lim_{x\to 0} \frac{\sin x \cos x}{x(1-x)}$$

If we try sandwich $\frac{\sin x \cos x}{x(x+1)}$ between $\pm \frac{1}{x(1-x)}$ we get the two outside limits diverge, This approach clearly doesn't work.

We do know $\lim_{x\to 0} \frac{\sin x}{x} = 1$ and $\lim_{x\to 0} \frac{\cos x}{1-x} = \frac{\lim_{x\to 0} \cos x}{\lim_{x\to 0} 1-x} = 1$

We can just use $\lim ab = \lim a \lim b$ (if both exist!)

So
$$\lim_{x\to 0} \frac{\sin x}{x}\frac{\cos x}{1-x} = \lim_{x\to 0}\frac{\sin x}{x} \lim_{x\to 0}\frac{\cos x}{1-x} = 1\cdot 1 = 1$$

20) $$\lim_{x\to 0} \frac{\csc x - \cot x}{x \csc x}$$

First, lets manipulate the eq a bit

$$\lim_{x\to 0} \frac{\frac{1}{\sin x} - \frac{\cos x}{\sin x}}{x\frac{1}{\sin x}} = \lim_{x\to 0}\frac{1-\cos x}{x} = 0$$

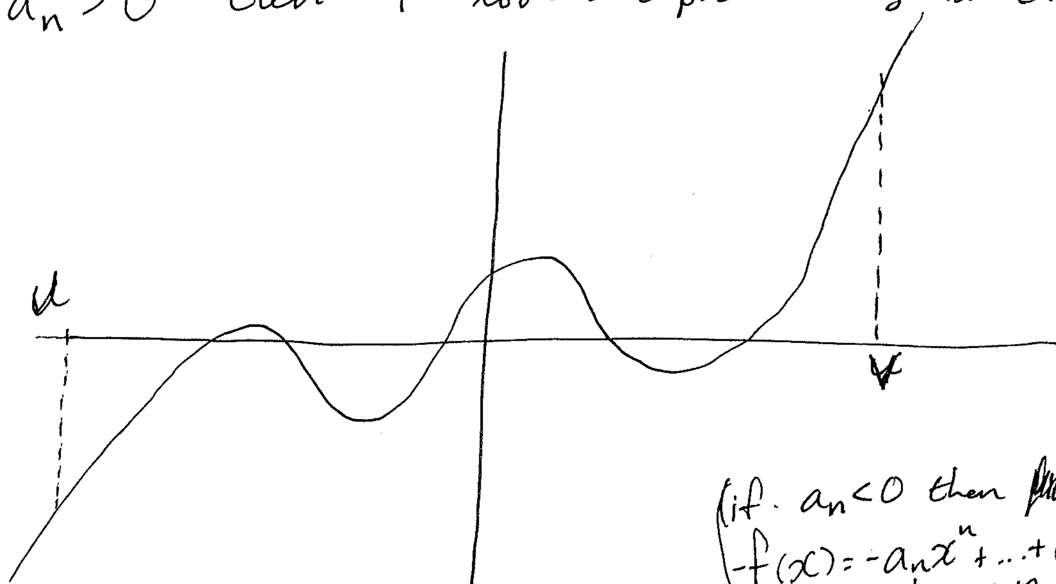
Show that odd degree polynomials have at least one root

To make things easier to say, let n be odd and

$f(x)=a_nx^n+a_{n-1}x^{n-1}+\dots+a_0$

This enables us to pick f appart more easily

If $a_n>0$ then f looks aproximatly like:



(if $a_n<0$ then consider $-f$ *

$-f(x)=-a_nx^n+\dots+0$ and $-a_n>0$

we have not changed the roots)

The area in the middle is not importaint, we care about the tails

The way we talk about them is through limits:

$$\lim_{x\to\infty} f(x)=\infty \quad \text{as } a_n>0$$

$$\lim_{x\to-\infty} f(x)=-\infty \quad \text{as } a_n>0$$

oh, and f is cont.!

To use the IVT we need an interval $[u,V]$ st $f(u)<0<f(v)$ √

The statement of the IVT requires an interval

As $\lim_{x\to\infty} f(x)=\infty$ we know $\exists\, v$ st $f(v)>0$ and $v>0$

as $\lim_{x\to-\infty} f(x)=-\infty$ we know $\exists\, u$ st $f(u)<0$ and $u<0$

So our interval is $[u,v]$, $\underbrace{\ }_{\text{by ivt}}$ $\exists\, y\in[u,v]$ st $f(y)=0$

To make this argument 100% mathematically precise we have to show

$$\lim_{x \to \infty} f(x) = \infty \quad \text{for } a_n > 0$$

also What if $a_n < 0$ (expanding on the aside)

if $a_n < 0$ then $-f(x) = -a_n x^n - a_{n-1} x^{n-1} + \ldots - a_0$

$$= b_n x^n + b_{n-1} x^{n-1} + \ldots + b_0$$

and we know $b_n > 0$, so we can apply the same proof without change to $-f(x) = g(x) = b_n x^n + \ldots + b_0$

It is common in math to solve it one way, then reduce the other cases to that one proof.