Solutions to Graded Problems
Homework # 1

Assignment

Section 1.1: 4, 28, 32, 46, 52, 62, 74, 78, 92, 106
1.2: 16, 18, 26, 29, 44, 58
1.3: 2, 8, 12, 16, 22, 30, 32.

1.1.52 The Celsius scale is devised so that $0°C$ is freezing temp of water and $100°C$ is the boiling pt. The Fahrenheit scale is set at $32°F$ and $212°F$, respectively.

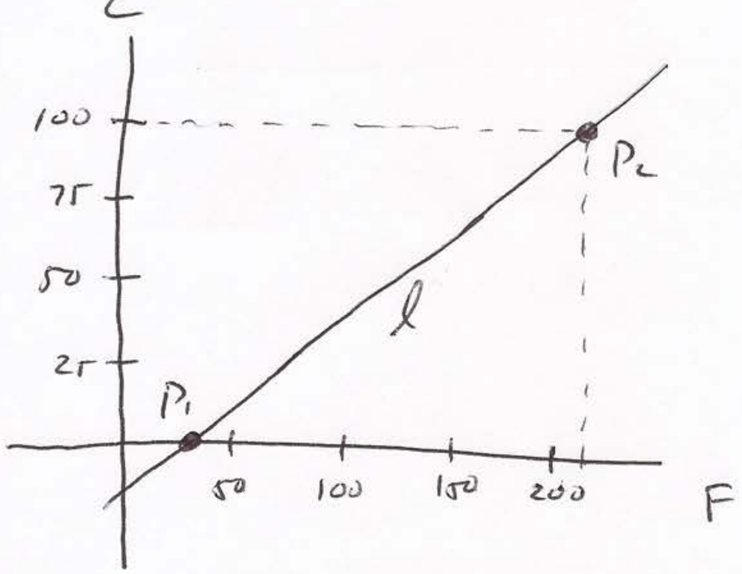
(a) Find the linear relationship between the 2 scales.

Strategy: Choose which of the 2 scales will be represented by the independent variable. Then create the 2 ordered pairs of numbers based on this choice and use them to construct the line that passes through them.

Solution: Let F be the independent variable (°F) and C the dependent variable (°C). Then the 2 ordered pairs are:

freezing temp: $(32, 0) = P_1 = (F_1, C_1)$

boiling temp: $(212, 100) = P_2 = (F_2, C_2)$



Slope of line ℓ is

$$m_\ell = \frac{C_2 - C_1}{F_2 - F_1} = \frac{100 - 0}{212 - 32} = \frac{5}{9}.$$

Then equation of ℓ is, using pt-slope formula is.

$$C - C_1 = m_\ell (F - F_1)$$

$$C - 0 = \frac{5}{9}(F - 32)$$

linear equation is:

$$\boxed{C = \frac{5}{9}(F - 32)}$$

Note: if done using C as the independent var. then linear equation is

$$F = \frac{9}{5}C + 32.$$

1.1.52 (b) Normal body temp ranges from 97.6°F to 99.6°F. Convert to Celsius.

~~Solution~~ Strategy: Solve for each edge of range the equivalent Celsius temp and form range.

Solution: Here linear eqn is $C = \frac{5}{9}(F-32)$

Lower edge is $C = \frac{5}{9}(97.6-32)$

$\cong 36.4$

Upper edge is $C = \frac{5}{9}(99.6-32)$

$\cong 37.6$

Hence converted range is from 36.4°C to 37.6°C.

1.1.62 Find the center and the radius of the circle $x^2+y^2+2x-4y+1=0$.

Strategy: Complete the square on each variable to write the equation as the sum of 2 perfect squares:

$$(x-a)^2+(y-b)^2=c^2$$

Then center is @ (a,b), and radius is c.

Solution: Rewrite equation as

$$(x^2+2x)+(y^2-4y) = -1.$$

Seeing that $(x+1)^2 = x^2+2x+1$

$(y-2)^2 = y^2-4y+4$, we add appropriately to both sides:

$$\underbrace{(x^2+2x+1)}+(y^2-4y+4) = -1+1+4$$

$$(x+1)^2+(y-2)^2=4.$$

Hence $(a,b)=(-1,2)$, $c=2$ and center is at $(-1,2)$ with radius 2. ∎

<u>1.2.16</u> Given $f(x) = \frac{1}{x+1}$, $x \neq 1$, and $g(x) = 2x^2$, $x \in \mathbb{R}$, find ⓐ $(f \circ g)(x)$, ⓑ $(g \circ f)(x)$ and their domains.

<u>Strategy</u>: Compose the functions and test for domain problems.

<u>Solution</u>: ⓐ $(f \circ g)(x) = f(g(x)) = \frac{1}{g(x)+1}$

$$= \frac{1}{2x^2+1}$$

The domain of $g(x)$ is all of $\mathbb{R}$. Hence we only need find if any input for $g(x)$ has output -1. Here $g(x) = -1 = 2x^2$ has no solutions.

Domain is all $\mathbb{R}$.

ⓑ $(g \circ f)(x) = g(f(x)) = 2(f(x))^2 = \frac{2}{(x+1)^2}$. Domain is all $x \neq 1$, the original domain of inside func $f(x)$. ∎

1.2.44 Growth model $r(N)$ is

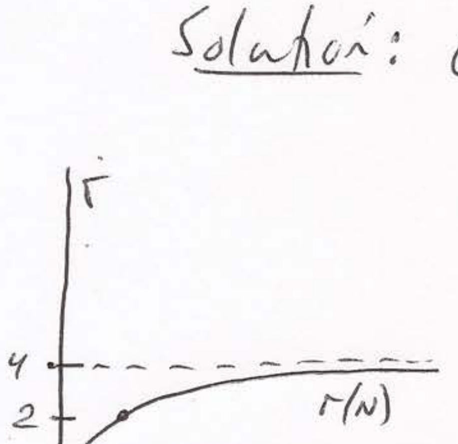
$$r(N) = a\frac{N}{k+N},\ N \geq 0$$

and models concentration of nutrients N and growth of a population r as a funct of N. Here a, k are positive constants.

(a) What happens to $r(N)$ as N increases? Why is a called the saturation level?

Strategy: Graph $r(N)$ for a few representative values of a, k and describe.

Solution: Choose $a=4$, $k=2$. Then $r(N)$ is



an increasing function that approaches 4. Using other choices of a, k, we find $r(N)$ is an increasing function that approaches a as N gets very large.

$r(N)$ approaches a but never passes it. This is why a is called the saturation level.

1.2.44 (b) k is called the half-saturation constant. Show if $N = k$, then $r(N) = \frac{a}{2}$.

Solution: Directly $r(k) = a\,\frac{k}{k+k} = a\left(\frac{k}{2k}\right) = \frac{a}{2}$. ∎

1.2.58 Assume a population N at time t is

$$N(t) = 40 \cdot 2^t, \quad t \geq 0.$$

(a) Find the population @ $t=0$.

Solution: $N(0) = 40 \cdot 2^{(0)} = 40$

The population @ $t=0$ is 40.

(b) Show $N(t) = 40e^{t\ln 2}$, $t \geq 0$.

Strategy: Convert the exponential 2^t to base e.

Solution: To convert to base e, we solve $2 = e^x$ for x. Converting $2 = e^x$ to a log equation, we get $2 = e^x \iff x = \log_e 2 = \ln 2$.

Hence $2^t = (e^{\ln 2})^t = e^{t\ln 2}$ and $N(t) = 40 \cdot 2^t = 40 \cdot e^{t\ln 2}$.

1.2.58 (c) How long until pop of needles 1000?

Strategy: Solve $N(t) = 1000$ for t.

Solution: (i) Using a calculator, we can solve

$$1000 = 40e^{t\ln 2}$$

$$25 = e^{t\ln 2} \underset{\text{conversion}}{\Longleftrightarrow} t\ln 2 = \ln 25$$

So that $t = \frac{\ln 25}{\ln 2}$. (Stop here without a calculator

$t \cong 4.64$ (no units)

(ii) Also, $1000 = 40 \cdot 2^t$

$$25 = 2^t \underset{\text{convert}}{\Longleftrightarrow} t = \log_2 25$$

and since $2^4 = 16 < 25 < 32 = 2^5$

we know $4 < t < 5$. One could

guess here that $t \cong 4.5$?

1.3.32 How to get from $y = \cos x$ to the following by basic transformations.

(a) $y = 1 + 2\cos x$

Solution: Take $y = \cos x$ and scale graph by 2 and shift up by 1 the result.

(b) $y = -\cos\left(x + \frac{\pi}{4}\right)$

Solution: First, we shift $y = \cos x$ to the left by $\frac{\pi}{4}$, then we multiply graph by -1, inverting it across the x-axis.

(c) $y = -\cos\left(\frac{\pi}{2} - x\right)$

Solution: First, note that $y = \cos x$ is an even function. Hence $\cos(-x) = \cos x$.
Hence $\cos\left(\frac{\pi}{2} - x\right) = \cos\left(x - \frac{\pi}{2}\right)$.
First, shift $y = \cos x$ by $\frac{\pi}{2}$ to the right.
Then multiply by -1, ~~[illegible]~~