5.3

#11) $y = e^{-|x|}$

This is an even function ie $f(-x) = f(x)$

This means anything I find out for $x > 0$, $(0, \infty)$

also holds (with a negation) for $x < 0$ $(-\infty, 0)$

This does not help me that much with it though

Rewrite y to not have abs. val.

$$y = \begin{cases} e^{-x} & x \geq 0 \\ e^{x} & x < 0 \end{cases}$$

$$\frac{dy}{dx} = \begin{cases} -e^{-x} & x > 0 \\ e^{x} & x < 0 \end{cases}$$

not defined at 0

$$-1 = \lim_{x \to 0^+} \frac{dy}{dx} \neq \lim_{x \to 0^-} \frac{dy}{dx} = 1$$

0 is the only critical point

Second derivative test is useless, I will use the ~~number line~~ table

	-1	0	1
$f(x)$	e^{-1}	1	$\frac{1}{e}$
$f'(x)$	$\frac{1}{e}$	0	$-\frac{1}{e}$

+ 0 − indicates local max

as $e^{-x} < 1$ for $x \in (0, \infty)$ we know its global

$f(x)$ is increasing for $x<0$ decreasing for $x>0$ and the global max is $(0,1)$. there is no global or local min

(21)

$f(x) = e^{-x^2}$ determine all inflection points

$f'(x) = -2xe^{-x^2}$

$f''(x) = -2e^{-x^2} + 4x^2e^{-x^2}$

$f''(x) = 0 = -2e^{-x^2} + 4x^2e^{-x^2}$

$e^{-x^2} \neq 0$ anywhere so

$f''(x) = 0 = 4x^2 - 2$

$x^2 = \frac{1}{2}$

$x = \frac{\pm 1}{\sqrt{2}}$

	-1	$\frac{-1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{2}}$	1
$f''(x)$	$\frac{4-2}{e}$	0	-2	0	$\frac{2}{e}$
	$+$	0	$-$	0	$+$

both are inflection points

22) Find two positive numbers a, b st $a+b=20$ and ab is at a maximum

I need to get rid of a variable, I choose b

$$a+b=20$$
$$b=20-a$$

So I need to maximize $ab = a(20-a)$

$$f(a) = a(20-a)$$
$$f'(a) = 20-2a$$
$$f'(a) = 0 = 20-2a$$
$a=10$ only critical point

	5	10	15
f	—	100	—
f'	10	0	-10
	+	0	−

so $a=b=10$ is a max with $ab=100$

6)

$$\lim_{x \to 0} \frac{3 - \sqrt{2x+9}}{2x}$$

$$\left.\begin{array}{l} \lim_{x \to 0} 3 - \sqrt{2x+9} = 3 - \sqrt{9} = 0 \\ \lim_{x \to 0} 2x = 0 \end{array}\right\} \text{so } \frac{0}{0}$$

$$\lim_{x \to 0} \frac{3 - \sqrt{2x+9}}{2x} = \lim_{x \to 0} \frac{\frac{2}{2\sqrt{2x+9}}}{2} = \lim_{x \to 0} \frac{1}{2\sqrt{2x+9}} = \frac{1}{6}$$

58) $$\lim_{x \to 0^+} \left(\frac{-1}{\sqrt{x}} + \frac{1}{x} \right)$$

$$\lim_{x \to 0^+} \frac{1}{x} = \infty$$

so $\infty - \infty$

$$\lim_{x \to 0^+} \frac{-1}{\sqrt{x}} = -\infty$$

$$\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{\sqrt{x}} \right) = \lim_{x \to 0^+} \frac{1 - \sqrt{x}}{x}$$

$$\lim_{x \to 0^+} x = 0^+$$

$$\lim_{x \to 0^+} 1 - \sqrt{x} = 1^-$$

No need for l'Hospital

$$\lim_{x \to 0^+} \frac{1 - \sqrt{x}}{x} = \infty$$ diverges!