1. Topics and connections for the topological Langlands seminar.

(1) Introduction to L-functions and ζ-functions.
  - (Dedekind) ζ-functions of number fields.
  - ζ-functions of varieties.
  - Artin L-functions.
  - Automorphic L-functions.
  - Langlands correspondences at the level of L-functions.

(2) Introduction to étale cohomology and its application as a Weil cohomology.
  - Lefschetz trace formulas.
  - The special case of an elliptic curve.
  - Galois cohomology.
  - Connection to topic 1: ζ-functions of varieties over finite fields.

(3) Introduction to algebraic K-theory.
  - What impact does $K_*(R)$ have on varieties over Spec R?
  - Connection to topic 1: Iwasawa theory and p-adic L-functions realized on $\widetilde{K_0(\mathcal{O}_K)}$.
  - Connection to topic 2: Quillen-Lichtenbaum conjecture.

(4) Introduction to $\mathbb{F}_1$.
  - Archimedean primes and analytic compactifications of Spec $\mathbb{Z}$.
  - Berkovich space.
  - Curves over Spec $\mathbb{F}_1$.
  - Connection to topic 1: Deformations of curves over Spec $\mathbb{F}_1$ as geometric realization spaces for Dedekind ζ-functions.
  - Connection to topic 2: The Galois group $G_{\overline{\mathbb{F}_1}/\mathbb{F}_1}$ and $H^*_{\acute{e}t}(\text{Spec } \mathbb{F}_1; \mathcal{F})$.
  - Connection to topic 3: Finite sets, $GL_n(\mathbb{F}_1) \cong \Sigma_n$, and $K_*(\mathbb{F}_1) \cong \pi_* B\Sigma_\infty^+$.

(5) Introduction to stable homotopy.
  - Stable homotopy of spheres.
  - E-Adams/descent spectral sequences; interpretation on derived stacks.
  - Morava E-theory and the stabilizer groups $\mathbb{S}_n$.
  - Connection to topic 1: Adams' result on $\zeta(-n)$ and the ANSS 1-line, generalizations.
  - Connection to topic 2: Crystalline cohomology as Weil cohomology, and crystalline representations of $\mathbb{S}_n$ on $(E_n)_*X$.
  - Connection to topics 3,4: QS^0 and $K_*(\mathbb{F}_1) \cong \pi_* S$.

(6) Introduction to class field theory.
  - Local class field theory over number fields, i.e., Lubin-Tate theory.
  - Global class field theory over number fields.
  - Class field theory over function fields.
  - Drinfeld's approaches to a Langlands correspondence for GL_2.
  - Connection to topic 1: Artin reciprocity as $n = 1$ local Langlands; geometric Langlands for GL_1.
  - Connection to topic 2: Brauer group as torsion in $H^2_{\acute{e}t}(X, \mathbb{G}_m)$, étale cohomology interpretation of proof of Artin reciprocity by Galois cohomology.
  - Connection to topic 3: Global class field theory and the class group of a number field.

- Connection to topic 4: Local fields with residue field $\mathbb{F}_{1^n}$ and their local class field theory; $W(\mathbb{F}_{1^n})$ and Lubin-Tate theory over $\mathbb{F}_1$.
- Connection to topic 5: Morava E_n as derived Lubin-Tate space; the Hopkins-Miller theorem and the groups acting on (the Drinfeld cover of) Lubin-Tate space; topological Hecke operators.

(7) Introduction to Shimura varieties.
- Hodge structures.
- The Siegel upper half-plane.
- Abelian varieties.
- Connection to topic 1: Shimura varieties in Langlands correspondences, the Harris-Taylor proof.
- Connection to topic 2: The groups acting on Sh and on $H^*_{\acute{e}t}(\mathrm{Sh}; \mathcal{F})$.
- Connection to topic 3: ?
- Connection to topic 4: Shimura varieties and lattices at Archimedean primes.
- Connection to topic 5: TAF as derived Shimura stack.
- Connection to topic 6: Shimura stacks and (Drinfeld covers of) Lubin-Tate space; Carayol's program and non-abelian Lubin-Tate theory.

As a rough guess, I would imagine that we could have one week of introduction to each topic, and then each connection to each other topic will take at least one additional week. If we find that one topic or connection or idea or project seems especially promising as we are talking about it in the seminar, then of course we should adjust the schedule to spend as much more time on it as we see fit.

2. Potential projects for the topological Langlands seminar.

- We should write up our notes for each talk.
- Eugene had the idea that we could work out a Langlands correspondence for local function fields. I think this would help ease the transition from number fields to function fields, and I think it would give some very useful hands-on practice.
- Eugene and Katia mentioned the idea of working out a theory of Witt vectors for $\mathbb{F}_1$. This seems very important to me also, since as I understand it, Spf $W(\mathbb{F}_{1^n})$ should be the natural base scheme for the geometric realization spaces for the Riemann and Dedekind ζ-functions, i.e., deformations of curves over Spec $\mathbb{F}_1$.

  The Witt ring $W(\mathbb{F}_{p^n})$ plays a central role in Lubin-Tate theory and stable homotopy as well, as $W(\mathbb{F}_{p^n})[[u_1, \ldots, u_{n-1}]]$ is the moduli ring of deformations of the Honda (universal) height n formal group law over $\mathbb{F}_{p^n}$. Assuming that some notion of an elliptic curve over Spec $\mathbb{F}_1$ can be worked out—which would be of great importance in itself, for studying the Riemann ζ-function—what would be its associated formal group over $\mathbb{F}_1$? If there is a theory of formal groups over $\mathbb{F}_1$, can we classify their deformations to characteristic zero using some kind of Lubin-Tate algebra over $W(\mathbb{F}_{1^n})$, and then recover a kind of local class field theory over $\mathbb{F}_1$?
- Jack and I have talked a little bit about the quotient space $\mathbb{C}//\mathbb{C}^\times$, which Kapranov showed was the moduli space of real Hodge structures, and the Connes adele class spaces $A//k^\times$. In each case one considers some additive object and quotients out by a multiplicative object; the analogue

for formal groups would be something like $\mathbb{G}_a//\mathbb{G}_m$. This is very nearly a thing familiar to people in computational stable homotopy: the group $\mathbb{G}_m(\hat{\mathbb{Z}}_p) \cong \hat{\mathbb{Z}}_p^\times \cong \mathrm{Aut}(F_1) \cong \mathbb{S}_1$ acts on the power series ring $\hat{\mathbb{Z}}_p[u]$, which is (part of) the representing object for $\mathbb{G}_a$ over Spf $\hat{\mathbb{Z}}_p$; the cohomology $H^*(\mathbb{G}_m(\hat{\mathbb{Z}}_p); \hat{\mathbb{Z}}_p[u])$ is (very close to) the E_2-term of a spectral sequence converging to $\pi_* L_{K(1)}S$, the stable homotopy of a particular important localization of the sphere.

In some loose sense we can think of the Morava E-theory E_1 as a derived representing object for a derived additive formal group, although this isn't true for higher heights $n > 1$; and the action of $\mathbb{G}_m(\hat{\mathbb{Z}}_p)$ lifts to E_1. The spectral sequence above is supposed to take us from the naive, algebraic group cohomology $H^*(\mathbb{G}_m(\hat{\mathbb{Z}}_p); \pi_*(E_1))$, i.e., derived functors of ordinary fixed-points, to the derived functors of the homotopy fixed-points, $E_1^{h\mathbb{G}_m(\hat{\mathbb{Z}}_p)}$ (which is homotopy equivalent to $L_{K(1)}S$, as a corollary of the Hopkins-Miller theorem). This is an example of an Adams spectral sequence, which I will write more about, below. The upshot is, again very loosely, that the Adams spectral sequence takes us from a (this is possibly a terrible name) "formal local Connes adele class space" $\mathbb{G}_a//\mathbb{G}_m$ to a (possibly an even worse name) "derived formal local Connes adele class space" $E_1//\mathbb{S}_1 \simeq L_{K(1)}S$. (9/19/2008 addendum: The variance on this entire construction is off: the homotopy fixed points $E_1^{h\mathbb{S}_1}$ map into E_1, while a derived class space construction $E_1//\mathbb{S}_1$ should map out of E_1. Perhaps the homotopy coinvariants $(E_1)_{h\mathbb{S}_1}$ are a closer analogue to the Connes adele class space.)

- Perspectives on Adams spectral sequences: there is the classical version which stable homotopy theorists have used for decades; Jack and I have talked about several re-interpretations of what the Adams spectral sequence does. It seems to describe fibrant replacement or "∞-sheafification" for global sections on a derived stack, it seems to be a kind of Leray spectral sequence for projection to Spec S (Jack's perspective), and it seems to interpolate between Galois cohomology of Tannakian group(oid)s of localizations of the stable homotopy category and a kind of associated filtration on the unit object of the monoidal structure on those localizations (also more or less Jack's perspective, I think).
- I would like to spend some time making sense of the maps between Shimura stacks and Drinfeld covers of Lubin-Tate spaces (i.e., moduli of deformations of formal groups with level structures), and trying to determine whether these maps lift to the derived setting, i.e., what are the universal maps between E_n and the spectra of topological automorphic forms.