PRACTICE PROBLEM

Let \( H \) the spiral staircase
\[
  x = u \cos \theta, \quad y = u \sin \theta, \quad z = \theta, \quad 0 \leq u \leq 1, \quad 0 \leq \theta \leq 2\pi
\]
(You might take a look at the picture on p. 464.)
a) Parametrize the boundary curve \( \partial H \). The boundary has four parts
\[
  \partial H = \gamma_1 + \gamma_2 - \gamma_3 - \gamma_4
\]
corresponding to
\[
  \theta = 0, \quad r = 1, \quad \theta = 2\pi, \quad u = 0
\]
respectively. Pay attention to the orientation of the boundary. (Note that \( H \) is parametrized by the box \([0, 1] \times [0, 2\pi]\) in the \((u, \theta)\)-plane. The boundary of \( H \) corresponds to the boundary of that box.)

2) Consider the vector field \( \vec{F} = x \hat{i} - y \hat{j} \). Prove that \( \vec{F} = \text{curl} \vec{G} \), with \( \vec{G} = x y \hat{k} \).
(why does such a vector field \( \vec{G} \) exist?)

3) Compute the flow of \( \vec{F} \) through \( H \) by using 2) and Stokes theorem.

4) Find a scalar function (potential) \( \phi(x, y, z) \) such that \( \nabla \phi = \vec{F} \) (why does such a potential exist?)

5) Compute the circulation of \( \vec{F} \) around the oriented boundary \( \partial H \).