PRACTICE PROBLEMS FOR MIDTERM 2

1. Integrals

1.1. Compute the volume of the sphere of radius $R$ by computing the integral
\[ \int \int_D (0,R) \sqrt{1 - x^2 - y^2} \, dx \, dy \] using polar coordinates. Here $D(0,R)$ is the disk $x^2 + y^2 \leq R^2$.

1.2. Compute the integral
\[ \int \int_D \frac{dx \, dy}{\sqrt{x^2 + y^2}} \] where $D$ is the domain in $\mathbb{R}^2$ bounded by the parabola $y = x^2$ and the line $y = x$.
Hint: use polar coordinates to parametrize the domain $D$ by $x = r \cos \theta, y = r \sin \theta$, with $0 \leq \theta \leq \frac{\pi}{4}$ and $0 \leq r \leq \frac{\sin \theta}{\cos \theta}$.

1.3. Compute
\[ \int \int_D x^2 \, y \, dx \, dy \] where $D$ is the domain in $\mathbb{R}^2$ bounded by the lines $y = 0, y = 1 - x$ and $y = x + 1$.

1.4. Compute
\[ \int \int_D xy^2 \sqrt{x^2 + y^2} \, dx \, dy \] where $D$ is the region of the disk $x^2 + y^2 \leq 1$ where $x \geq 0$ and $y \leq 0$.

1.5. [Ex. 10, p. 327] Find the volume bounded by the graph of $f(x, y) = 1 + 2x + 3y$, the rectangle $[1, 2] \times [0, 1]$ and the four vertical planes bounding the rectangle.

1.6. \[ \int \int_{[-1,1] \times [0,1]} (x^2 + y^2) \, dx \, dy. \]

1.7. Determine the volume of the region in $\mathbb{R}^3$ bounded by the paraboloid $z = 4 - x^2 - y^2$ and the $xy$-plane.

1.8. Calculate the volume bounded above by the graph $z = 1 - x^3 - y^3$ and below and on the sides by the coordinate planes $x = 0, y = 0, z = 0$. 

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