HW6 (2)

3. Compute the area under the parametric curve $x = e^t$, $y = t^2$ for $t \in [0,1]$

sol: $\text{Area} = \int_{t_1}^{t_2} y(t)\, x'(t)\, dt$ $\qquad x' = e^t$

$= \int_0^1 t^2 e^t\, dt$ $\qquad \begin{bmatrix} u = t^2 & dv = e^t dt \\ du = 2t\,dt & v = e^t \end{bmatrix}$

$= t^2 e^t \Big|_0^1 - 2\int_0^1 t e^t\, dt$ $\qquad \begin{bmatrix} u' = t & dv' = e^t dt \\ du' = dt & v' = e^t \end{bmatrix}$

$= t^2 e^t \Big|_0^1 - 2\left[t e^t \Big|_0^1 - \int_0^1 e^t dt \right]$

$= t^2 e^t - 2te^t + 2e^t \Big|_0^1$

$= e - \cancel{2e} + \cancel{2e} - \cancel{0} + \cancel{0} - 2 = \boxed{e-2}$

* 4. Compute the arc length of the curve $x = 2\sin t + 3$, $y = 2\cos t + 4$ for $t \in [0, \frac{\pi}{3}]$

sol: $L = \int_{t_1}^{t_2} \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$ ...... 2" $\qquad x'(t) = 2\cos t \qquad y'(t) = -2\sin t$ ... 1"

$= \int_0^{\frac{\pi}{3}} \sqrt{4\cos^2 t + 4\sin^2 t}\, dt$

$= \int_0^{\frac{\pi}{3}} 2\, dt = 2t \Big|_0^{\frac{\pi}{3}} = \boxed{\frac{2\pi}{3}}$. ... 2"

* 5. Find a polar equation for the curve represented by the Cartesian equation $y^2 = x$.

$y = r\sin\theta$ $\qquad (r\sin\theta)^2 = y^2 = x = r\cos\theta \Rightarrow r^2 \sin^2\theta = r\cos\theta$ ...... 1"

$x = r\cos\theta$ ...... 2" $\qquad \Rightarrow \boxed{r = \frac{\cos\theta}{\sin^2\theta} = \cot\theta \csc\theta}$ ...... 2"

6. Find a Cartesian equation for the curve represented by the polar equation $r^2 = 3\theta$

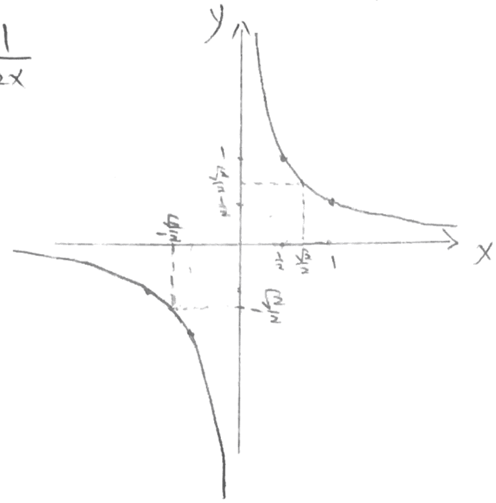
$r^2 = x^2 + y^2 \qquad \theta = \tan^{-1}\frac{y}{x} \qquad r^2 = 3\theta \Rightarrow x^2 + y^2 = 3\tan^{-1}\frac{y}{x}$

$\boxed{\tan\left(\frac{x^2+y^2}{3}\right) = \frac{y}{x}}$

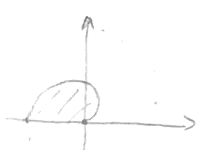
7. Sketch the polar curve $r^2 \sin 2\theta = 1$

$$r^2 \sin 2\theta = r^2\, 2 \sin\theta \cos\theta = 2r^2 \frac{y}{r} \cdot \frac{x}{r} = 2xy = 1$$

$$y = \frac{1}{2x}$$



* 8. $r = 3\theta$ is a polar curve. Compute the area of the polar region for $\theta \in [0, \pi]$.



$$A = \int_{\theta_1}^{\theta_2} \frac{1}{2} r^2 \, d\theta = \int_0^{\pi} \frac{1}{2} 9\theta^2 \, d\theta = \frac{3}{2}\theta^3 \Big|_0^{\pi} = \boxed{\frac{3}{2}\pi^3} \text{......} 2"$$

..... 2" ...... 1"