Example 2. $y' = x^2 + y^2 - 1$.

- Draw a vector (line segment) with slope $F(x, y)$ at each point $(x, y)$. Then connecting curves are approximation of the integral curves.

- The denser the chosen points are distributed on the plane, the closer the connecting curves are to the actual integral curves.
9.2: Direction fields and Euler method

- This gives numerical approximation of the integral curve.
- It is easy to draw integral curves by connecting direction vectors. Actual solutions are too hard to find.
Euler’s method:

\[ y_1 = y_0 + hF(x_0, y_0), \]
\[ y_2 = y_2 + hF(x_1, y_1), \]
\[ \ldots \ldots \ldots \]
\[ y_n = y_{n-1} + hF(x_{n-1}, y_{n-1}). \]

See picture for \( y' = y - x \) on \([2, 4]\).
9.3: Separable Equations

An equation is separable if one can use algebra to separate the two variables, so that each side of the equation only contains 1 variable.

Example 1. \( y' = x(y - 1) \)

Solution: Rewrite the equation as

\[
\frac{dy}{dx} = x(y - 1).
\]

\( y = 1 \) is a solution. Suppose \( y \neq 1 \), then we separate the variables:

\[
\frac{dy}{y - 1} = xdx.
\]
Example 1

Integrate both sides,

\[ \int \frac{dy}{y - 1} = \int x \, dx. \]

\[ \Rightarrow \ln |y - 1| + C_1 = \frac{x^2}{x} + C_2. \]

\[ \Rightarrow \ln |y - 1| = \frac{x^2}{x} + C_3. \]

Thus \(|y - 1| = e^{C_3} \cdot e^{\frac{x^2}{2}}.\)

\[ \Rightarrow y = 1 \pm e^{C_3} e^{\frac{x^2}{2}}, \quad \text{for all constants } C_3. \]

\[ \Rightarrow y = 1 + C_4 e^{\frac{x^2}{2}}, \quad \text{for all constants } C_4. \]