AS.110.109: Calculus II (Eng)

Chapter 10: Parametric Equations and Polar coordinates

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Parametric curves

Suppose that x, y are both given as functions of a third variable t (called a parameter)

$$x = f(t), y = g(t),$$

where $t \in (a, b)$.

- Parametric equations.
- As t varies, the collection of points $(x(t), y(t))$ form a curve. We call it parametric curve.

Parametric curves

- Some parametric curves can be written in Cartesian equation (i.e. uses only x and y without introducing the parameter t).

  Example 1. $x = \cos t, y = \sin t, 0 \leq t \leq 2\pi$ is a parametric curve.

  Indicate with an arrow the direction in which the curve is traced as the parameter increases.

  Eliminate the parameter to find a Cartesian equation of the curve.

  Indicate with an arrow the direction in which the curve is traced as the parameter increases.

Parametric curves

- Solution: Our goal is to eliminate the variable t.
  Each point $(x(t), y(t))$ satisfies

  $$x^2 + y^2 = 1.$$

  Thus it is on the circle. Also, every point on the circle corresponds to a point $(x(t), y(t))$ for some $t \in [0, 2\pi]$.

Parametric curves

In some cases, we can also transform the Cartesian equation to the parametric equations.

- Example 2. Write the parabola $y = x^2$ in parametric equations in t.
  Remark: There are many parametric equations that satisfies $y = x^2$. We only need to find one of them.
  Solution: $x = t$, $y = t^2$, $t \in (-\infty, \infty)$.

Parametric curves

- Example 3. $x = \frac{1}{2}\cos\theta, y = 2\sin\theta$, $0 \le \theta \le \pi$. Sketch the curve and indicate with an arrow the direction in which the curve is traced as the parameter increases.

Parametric curves

- Solution:
  - Notice
$$4x^2 + \frac{y^2}{4} = 1.$$
This is equation of an ellipse. See the picture.
  - Since $0 \le \theta \le \pi$, $y \ge 0$.
This is not the whole ellipse, but only the upper half of the ellipse.

Parametric curves

- Example 4. Find the parametric equation for the circle centered at $(1, 2)$ with radius 2.
  Solution: $x = 1 + 2\cos t,\ y = 2 + 2\sin t,\ t \in [0, 2\pi]$.